

Coordinate Geometry: The Circle

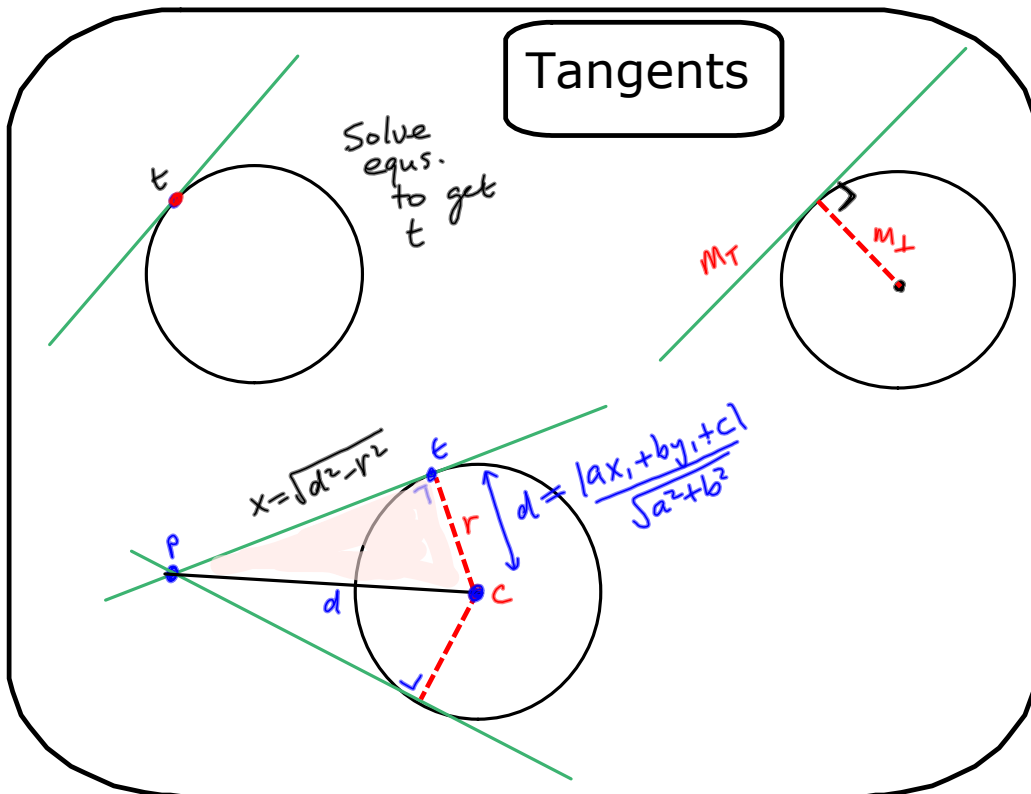
chapter

4

Section 4.4 Tangents to a circle

PROJECT MATHS – STRAND 2
Text & Tests 4
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1. Finding the equation of the tangent to a circle at the point P on the circle

Example 1

Find the equation of the tangent to the circle $x^2 + y^2 - 4x + 2y - 20 = 0$ at the point $(5, -5)$ on the circle.

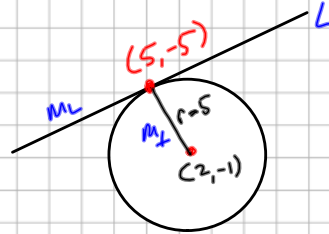
$$c = (2, -1)$$

$$r = \sqrt{2^2 + 1^2 + 20} = 5$$

$$L = ?$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



$$m_1 = \frac{-1 + 5}{2 - 5} = \frac{4}{-3} \quad \perp \quad \frac{3}{4} = m_L$$

$$L: y + 5 = \frac{3}{4}(x - 5)$$

$$4y + 20 = 3x - 15$$

$$L: \boxed{3x - 4y - 35 = 0}$$

1. Finding the equation of the tangent to a circle at the point P on the circle

Example 1

Find the equation of the tangent to the circle $x^2 + y^2 - 4x + 2y - 20 = 0$ at the point $(5, -5)$ on the circle.

Tangent to circle through given point: $(x - h)(x_1 - h) + (y - k)(y_1 - k) = r^2$ or $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

$$\text{Centre } (h, k) = (2, -1)$$

$$\text{Radius} = \sqrt{2^2 + 1^2 + 20} = 5$$

$$\text{Tangent pt } (x_1, y_1) = (5, -5)$$

$$\Rightarrow (x - 2)(5 - 2) + (y + 1)(-5 + 1) = 25$$

$$(x - 2)(3) + (y + 1)(-4) = 25$$

$$3x - 6 - 4y - 4 = 25$$

$$3x - 4y - 10 = 25$$

$$\boxed{3x - 4y - 35 = 0}$$

2 Tangents to a circle parallel or perpendicular to a given line

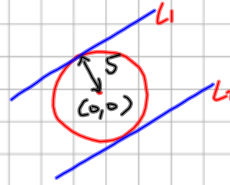
Example 2

Find the equations of the two lines parallel to the line $3x + 4y - 6 = 0$ and which are tangents to the circle $x^2 + y^2 = 25$.

Any line // to
 $3x + 4y - 6 = 0$

$$\Rightarrow 3x + 4y + k = 0$$

Centre $(0, 0)$
Radius $= 5$



distance from pt. to line:

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$\Rightarrow 5 = \frac{|3(0) + 4(0) + k|}{\sqrt{3^2 + 4^2}}$$

$$5 = \frac{|k|}{5}$$

$$25 = |k| \quad \Rightarrow k = 25 \text{ or } k = -25$$

equations of parallel lines: $L_1: 3x + 4y + 25 = 0$
 $L_2: 3x + 4y - 25 = 0$

2. Find the equation of the tangent to the circle $x^2 + y^2 = 10$ at the point $(-3, 1)$.

$$\text{Tangent: } -3x + 1y = 10$$

3. Find the equation of the tangent to the circle $x^2 + y^2 = 17$ at the point $(4, -1)$.



Tangent: $4x - y = 17$