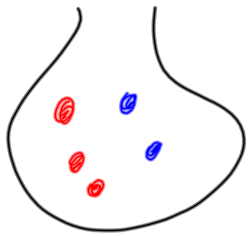
chapter 3 Probability 2

Section 3.4 How to show events are independent



Pick 2

Replace each time

Independent or Conditional?

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In Section 1.6 of this book, it was stated that two events are **independent** when the outcome of one event does not affect the outcome of the other event.

We also used the **multiplication rule** for independent events A and B which stated that

$P(\text{A and B}) = P(\text{A}) \times P(\text{B})$.

$P(\text{A and B}) = P(\text{A} \cap \text{B})$

Using set notation, this rule may be written as:

$P(\text{A} \cap \text{B}) = P(\text{A}).P(\text{B})$

Multiplication rule for independent events

In this section, we will use the multiplication rule to determine whether or not two events are independent.

The following examples will illustrate this procedure.

$P(A \cap B) = P(A).P(B)$ ← only true if independent

or

$P(A \cap B) = P(A).P(B|A)$ ← always true

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

If independent then:

① $P(A).P(B) = P(A).P(B|A)$

② $P(B) = P(B|A)$

If events A and B are independent then

$$P(A) = P(A|B)$$

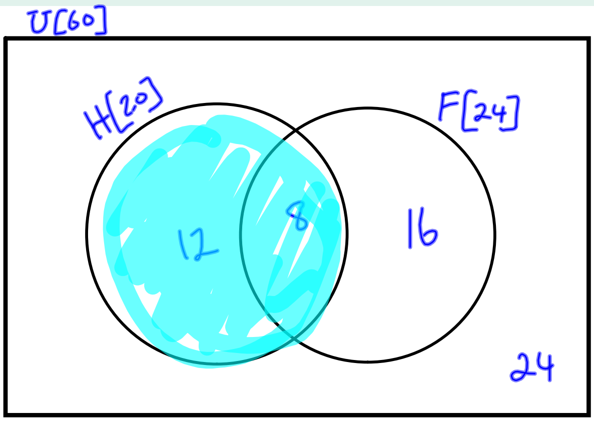
This is because P(AIB) is not effected by event B having happened because they are independent

also

$$P(B) = P(B|A)$$

Example 1

In a group of 60 students, 20 study History, 24 study French, 8 study both History and French and 24 study neither.
Illustrate this information on a Venn diagram.
Now investigate if the events 'a student studies History' and 'a student studies French' are independent.



Is $P(\cancel{H}).P(F) = P(\cancel{H}).P(F|H)$?

$P(H) = \frac{20}{60} = \frac{1}{3}$

$P(F) = \frac{24}{60} = \frac{2}{5}$

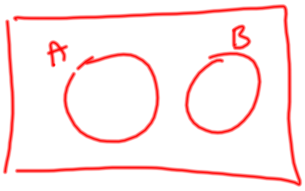
$P(F|H) = \frac{8}{20} = \frac{2}{5}$

$P(F) = P(F|H) \overset{\text{ie}}{\Rightarrow} \frac{2}{5} = \frac{2}{5}$ independent

Two rules that we have already dealt with are highlighted in the box opposite. They are used frequently when we are investigating whether or not two events are independent. These two rules should be memorised.

1. The addition rule:
   $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
2. Conditional probability: $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

Note: To show that two events A and B are mutually exclusive, we must show that
$P(A \text{ or } B) = P(A) + P(B)$.



Mutually exclusive is not to be confused with independence.

M.E. means $P(A \cap B) = 0$
ie.. both can not occur together.

Example 2

Two events A and B are such that $P(A) = 0.5$, $P(B) = 0.4$ and $P(A|B) = 0.3$.
(i) Find $P(A \cap B)$.
(ii) Investigate whether or not the events A and B are independent.

$P(A \cap B) = P(B).P(A|B)$

$P(A \cap B) = (0.4)(0.3)$
$= 0.12$

Independent?
Is $P(A) = P(A|B)$?

Since $P(A) = 0.5 \neq P(A|B) = 0.3$
$\Rightarrow$ not independent.

Example 3

Two ordinary fair dice, one red and one blue, are to be rolled once.

(i) Find the probability of the following events:

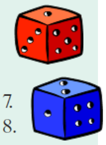
Event A: the number showing on the red dice will be a 5 or a 6.

Event B: the total of the numbers showing on the two dice will be 7.

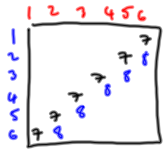
Event C: the total of the numbers showing on the two dice will be 8.

(ii) Show that events A and B are independent.

(iii) Investigate if events A and C are independent.



(i)



Independent?

Is $\cancel{P(A)}P(B) = \cancel{P(A)}.P(B|A)$?

(i) $P(A) = \frac{2}{6} = \frac{1}{3}$

$P(B) = \frac{6}{36} = \frac{1}{6}$

$P(C) = \frac{5}{36}$

(ii) Show A and B are independent

$P(B|A)$ ie.. P(total 7 given red is 5 or 6)

$= \frac{2}{12} = \frac{1}{6}$

Since $P(B) = \frac{1}{6} = P(B|A) = \frac{1}{6}$

$\Rightarrow$ Independent.

Is $P(C) = P(C|A)$?

$P(C|A)$ ie.. P(total 8 given red is 5 or 6)

$= \frac{2}{12} = \frac{1}{6}$

Since $P(C) = \frac{5}{36} \neq P(C|A) = \frac{1}{6}$

$\Rightarrow$ not independent.