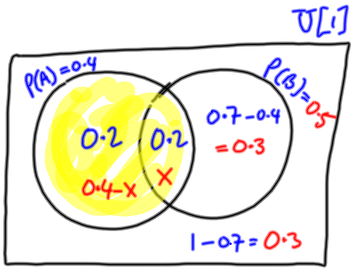
5. A and B are two independent events.
If $P(A) = 0.4$ and $P(A \cup B) = 0.7$, use the formula
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ to find $P(B)$.

Venn diagram



Independent !

$P(A \cap B) = P(A).P(B)$

$x = 0.4(0.3+x)$

$x = 0.12 + 0.4x$

$x - 0.4x = 0.12$

$0.6x = 0.12$

$x = \frac{0.12}{0.6} = 0.2$

Venn Diagram

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(B) = x + 0.3 = 0.2 + 0.3 = 0.5$

9. Given that events C and D are independent and that $P(C|D) = \frac{2}{3}$ and $P(C \cap D) = \frac{1}{3}$, find
(i) $P(C)$ (ii) $P(D)$.

"Independent"

$\Rightarrow P(C \cap D) = P(C).P(D)$

also

$\Rightarrow P(C|D) = P(C)$ (i) $\Rightarrow P(C) = \frac{2}{3}$

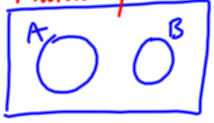
$\Rightarrow P(D) = \frac{P(C \cap D)}{P(C)}$ (ii) $\Rightarrow P(D) = \frac{1/3}{2/3} = \frac{1}{2}$

16. Two events, E and F, are such that $P(E) = \frac{2}{5}$, $P(F) = \frac{1}{6}$ and $P(E \cup F) = \frac{13}{30}$. Show that E and F are neither mutually exclusive nor independent.

If M.E. $\Rightarrow$

$P(A \cap B) = 0$

mutually exclusive



$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

$$\Rightarrow \frac{13}{30} = \frac{2}{5} + \frac{1}{6} - P(E \cap F)$$

$$\frac{13}{30} = \frac{17}{30} - P(E \cap F)$$

$$\Rightarrow P(E \cap F) = \frac{2}{15} \neq 0 \Rightarrow \text{not M.E.}$$

If independent $\Rightarrow$

$P(E \cap F) = P(E).P(F)$?

Is $\frac{2}{15} = \left(\frac{2}{5}\right)\left(\frac{1}{6}\right)$?

$$\frac{2}{15} \neq \frac{2}{30}$$

$\Rightarrow$ not independent.