Example 1

The numbers 1 to 9 are written on cards and placed in a box.
A card is drawn at random from the box.
Find the probability that the number is prime, given that the number is odd.

1 ✗ ~~2~~ 3 ✓ ~~4~~ 5 ✓ ~~6~~ 7 ✓ ~~8~~ 9 ✗

$$P(A \cap B) = P(A) * P(B|A)$$

$$\Rightarrow P(B|A) = \frac{P(A \cap B)}{P(A)}$$

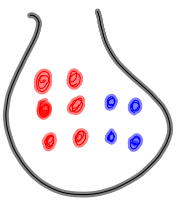
$$\boxed{\begin{aligned} P(\text{Prime} \cap \text{odd}) &= \frac{3}{9} \\ P(\text{odd}) &= \frac{5}{9} \end{aligned}}$$

$$P(\text{Prime}\,|\,\text{odd}) = \frac{\left(\frac{3}{9}\right)}{\left(\frac{5}{9}\right)} = \frac{3}{5}$$

Example 2

A bag contains 6 red and 4 blue discs. A disc is drawn from the bag and not replaced. A second disc is then drawn.
Find the probability that
(i) the first two discs are blue
(ii) the second disc drawn is red
(iii) one disc is red and the other disc is blue
(iv) both discs are the same colour.

NOT INDEPENDENT
→ CONDITIONAL PROBABILITY



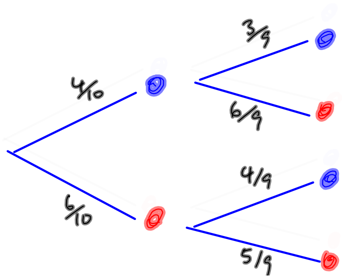
(i) $P(B.B) = \left(\frac{4}{10}\right)\left(\frac{3}{9}\right) = \frac{4}{30}$

(ii) $P(\text{Second is Red})$

$= P(RR \text{ or } BR) = \left(\frac{6}{10}\right)\left(\frac{5}{9}\right) + \left(\frac{4}{10}\right)\left(\frac{6}{9}\right) = \frac{3}{5}$

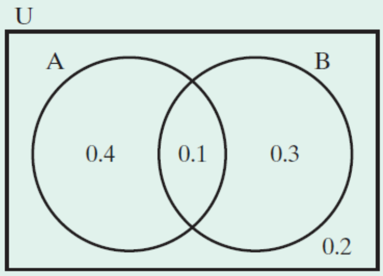
(iii) $P(RB \text{ or } BR) = \left(\frac{6}{10}\right)\left(\frac{4}{9}\right) + \left(\frac{4}{10}\right)\left(\frac{6}{9}\right) = \frac{8}{15}$

(iv) $P(RR \text{ or } BB) = \left(\frac{6}{10}\right)\left(\frac{5}{9}\right) + \left(\frac{4}{10}\right)\left(\frac{3}{9}\right) = \frac{7}{15}$



Example 3

Use the given Venn diagram to write down
(i) $P(A \mid B)$ (ii) $P(B \mid A)$



conditional probability

$P(A \cap B) = P(A) * P(A \mid B) \Rightarrow$ $P(A \mid B) = \dfrac{P(A \cap B)}{P(A)}$

(i) $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.1}{0.4} = \dfrac{1}{4}$

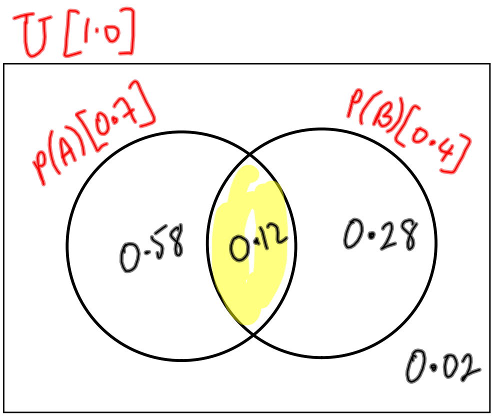
"A given B"

(ii) $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{0.1}{0.5} = \dfrac{1}{5}$

"B given A"

Example 4

Two events A and B are such that $P(A) = 0.7$, $P(B) = 0.4$ and $P(A \mid B) = 0.3$. Determine the probability that neither A nor B occurs.



conditional probability

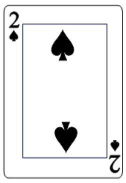
$P(A \cap B) = P(B) * P(A \mid B)$
$= (0.4)(0.3) = 0.12$

$P(A \cup B)' = 1 - P(A \cup B) = 0.02$

Exercise 1.7

1. A card is drawn at random from a pack of 52 playing cards.

(i) Given that the card is black, find the probability that it is a spade.
(ii) Given that the card is red, find the probability that it is a queen.
(iii) Given that the card is a picture card, find the probability that it is a king.



$P(\text{Spade} \mid \text{Black}) = \frac{1}{2}$

$P(\text{Queen} \mid \text{Red}) = \frac{2}{26} = \frac{1}{13}$

$P(\text{King} \mid \text{Picture}) = \frac{4}{12} = \frac{1}{4}$

2. The table shows information about a group of adults.

(i) A person is chosen at random from the group.
What is the probability that the person can drive?
(ii) A man in the group is chosen at random.
What is the probability that he can drive?
(iii) Find the probability that a person chosen at random can drive, given that the person is a female.

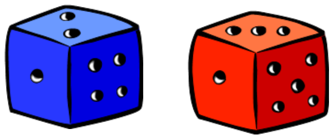
	Can drive	Cannot drive	
Male	32	8	40
Female	38	12	50
	70	20	90

(i) $P(\text{Can drive}) = 70/90 = 7/9$

(ii) $P(\text{Can drive} \mid \text{man}) = \frac{32}{40} = \frac{4}{5}$

(iii) $P(\text{can drive} \mid \text{female}) = \frac{38}{50} = \frac{19}{25}$

3. Two fair dice are thrown and the product of the numbers showing is recorded. Given that one dice shows a 2, find the probability that the product of the two numbers showing is
(i) exactly 6 (ii) 6 or more.



Sample space of products

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

$P(\text{Product } 6) = \frac{4}{36} = \frac{1}{9}$

$P(\text{Product } 6 \text{ or more}) = \frac{26}{36} = \frac{13}{18}$

4. A school enters 120 pupils for the Junior Certificate maths exam.
The given table shows the details of the entries.

	Ordinary	Higher	
Girls	20	35	55
Boys	25	40	65
	45	75	120

(i) Write down the probability that a pupil chosen at random is entered for Ordinary level.
(ii) A pupil is chosen at random. This pupil is a girl.
Find the probability that the girl was entered for Higher level.
(iii) A pupil is chosen at random. The pupil was entered for Ordinary level.
Find the probability that the pupil was a boy.

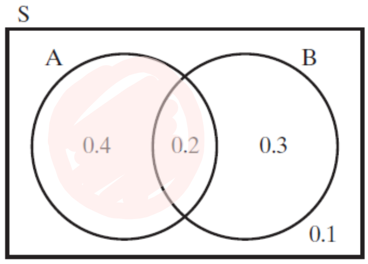
(i) $P(OL) = \frac{45}{120} = \frac{3}{8}$

(ii) $P(HL \mid G) = \frac{35}{55} = \frac{7}{11}$

(iii) $P(B \mid OL) = \frac{25}{45} = \frac{5}{9}$

12. Based on the probabilities shown in the given Venn diagram, find each of the following:

(i) $P(A)$
(ii) $P(A \cap B)$
(iii) $P(A \cup B)$
(iv) $P(A \mid B)$
(v) $P(B \mid A)$.



$P(A \cap B) = P(A) * P(B|A) \Rightarrow P(B|A) = \frac{P(A \cap B)}{P(A)}$

(i) $P(A) = 0.6$

(ii) $P(A \cap B) = 0.2$

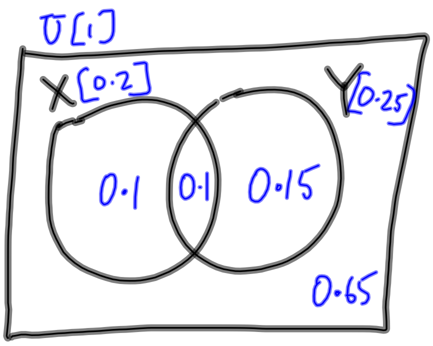
(iii) $P(A \cup B) = 0.9$

(iv) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.5} = \frac{2}{5}$

(v) $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.2}{0.6} = \frac{1}{3}$

14. X and Y are two events such that $P(X) = 0.2$, $P(Y) = 0.25$ and $P(X \cap Y) = 0.1$.
Illustrate this information on a Venn diagram.
Use the diagram to find
(i) $P(X \cup Y)$ (ii) $P(X \mid Y)$ (iii) $P(Y \mid X)$.

$P(A \cap B) = P(A) * P(B|A) \Rightarrow P(B|A) = \frac{P(A \cap B)}{P(A)}$



(i) $P(X \cup Y) = 0.35$

(ii) $P(X|Y) = \frac{P(X \cap Y)}{P(Y)}$

$= \frac{0.10}{0.25} = \frac{10}{25} = \frac{2}{5}$

(iii) $P(Y|X) = \frac{P(X \cap Y)}{P(X)}$

$= \frac{0.1}{0.2} = \frac{1}{2}$