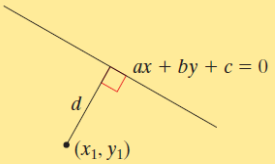
3. Tangents to a circle from a point P not on the circle

The perpendicular distance from the point (x_1, y_1) to the line $ax + by + c = 0$ is

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$



Example 3

Find the equations of the tangents to the circle $x^2 + y^2 = 5$ from the point $(5, 0)$.

Tangent = Line

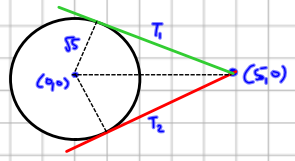
equation of line:
$y - y_1 = m(x - x_1)$
contains (5,0)

distance from (0,0) to tangent is R.

$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$ (R)

$a = m$
$b = -1$
$c = -5m$

Sub m values back into ①



$\Rightarrow y - 0 = m(x - 5)$

$y = m(x - 5) \Rightarrow mx - y - 5m = 0$ ①

$ax + by + c = 0$

$\sqrt{5} = \frac{|m(0) - 1(0) - 5m|}{\sqrt{m^2 + (-1)^2}}$

$\sqrt{5} = \frac{|-5m|}{\sqrt{m^2+1}} \Rightarrow \sqrt{5}\sqrt{m^2+1} = |-5m|$

$5(m^2 + 1) = 25m^2 \Rightarrow 5m^2 + 5 = 25m^2$

$5 = 20m^2 \Rightarrow \frac{1}{4} = m^2 \Rightarrow m = \pm\frac{1}{2}$

$T_1: (\frac{1}{2})x - y - 5(\frac{1}{2}) = 0$ (×2) $\Rightarrow x - 2y - 5 = 0$

$T_2: (-\frac{1}{2})x - y - 5(-\frac{1}{2}) = 0$ (×2) $\Rightarrow x + 2y + 5 = 0$