Coordinate Geometry: The Line

Chapter 1

Section 1.3 The equation of a line

2 common ways of presenting the equation of a line:

① Slope/intercept form

$y = mx + c$

slope y-intercept

② Standard form: $ax + by + c = 0$

$\boxed{m = \frac{-a}{b}}$ 'Read slope'

Find equation?

- $y - y_1 = m(x - x_1)$
- $y = mx + c$

$m = ?$
$pt = ?$

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Example 1

Find the equation of the line through the point $(-2, 3)$ which is perpendicular to the line $2x - y + 5 = 0$.

Plan:	
① • Find $m_\perp$ $m = -a/b$	① $m_1 = \frac{-2}{-1} = 2$
② • then line m $m_1 \times m_2 = -1$	② Slope of line? $m_\perp = 2 \quad \perp \quad -\frac{1}{2} = m$
③ • equation $y - y_1 = m(x - x_1)$ $\times 2$	③ equation $y - 3 = -\frac{1}{2}(x + 2)$ $2(y - 3) = -1(x + 2)$ $2y - 6 = -x - 2$
Standard form	$x + 2y - 4 = 0$

Example 2

Find the value of k if the lines $2x + ky + 5 = 0$ and $(k + 6)x + 2y - 9 = 0$ are perpendicular to each other.

Slope from equation:

$m = \frac{-a}{b}$

Line 1 : $2x + ky + 5 = 0$

$\boxed{m_1 = \frac{-2}{k}}$

Line 2 : $(k+6)x + 2y - 9 = 0$

$\boxed{m_2 = \frac{-(k+6)}{2}}$

Perpendicular slopes:

$m_1 \times m_2 = -1$ ⇒ $\left(\frac{-2}{k}\right)\left(-\frac{(k+6)}{2}\right) = -1$

$\times 2k$ $\quad 2(k+6) = -2k$

$2k + 12 = -2k$

$+2k, -12$ $\quad 4k = -12$

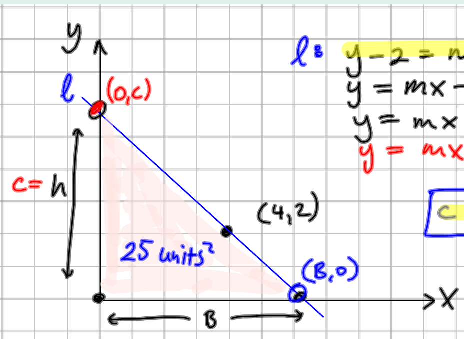
$\div 4$ $\quad ⇒ \boxed{k = -3}$

Example 3

Find the equations of the line l through the point $(4, 2)$ so that the area of the triangle formed by l and the positive x and y-axes is 25 square units.

$m = ?$

Diagram



equation:

$y - y_1 = m(x - x_1)$

l: $y - 2 = m(x - 4)$ ①

$y = mx - 4m + 2$

$y = mx + (2 - 4m)$

$y = mx + c$

$\boxed{c = 2 - 4m}$ ②

$(B, 0) \in l$

Sub into ①

⇒ $0 - 2 = m(B - 4)$

$-2 = Bm - 4m$

$Bm = 4m - 2$ ⇒ $\boxed{B = \frac{4m - 2}{m}}$ ③

$A = \frac{Bh}{2}$ ⇒ $25 = \frac{\left(\frac{4m-2}{m}\right)(2 - 4m)}{2}$ ⇒ $50 = \left(\frac{4m-2}{m}\right)(2 - 4m)$

$50m = 8m - 16m^2 - 4 + 8m$

$16m^2 + 34m + 4 = 0$

$8m^2 + 17m + 2 = 0$ ⇒ $(8m + 1)(m + 2)$

$m = -1/8$ or $m = -2$

17. The line $\ell_1: 3x - 2y + 7 = 0$ and the line $\ell_2: 5x + y + 3 = 0$ intersect at the point P. Find the equation of the line through P perpendicular to ℓ_2.

① Point? Point of intersection ⇒ Solve equations

$$3x - 2y = -7$$
$$5x + y = -3$$

$$\Rightarrow \quad 3x - 2y = -7$$
$$10x + 2y = -6$$
$$13x = -13$$
$$\boxed{x = -1}$$

$x = -1$

$$\Rightarrow 5(-1) + y = -3$$
$$-5 + y = -3 \quad \Rightarrow \boxed{y = 2}$$

Intersection $P(-1, 2)$

② Slope? Slope from equation $m = \frac{-a}{b}$

$$m_2 = \frac{-5}{1} \Rightarrow \boxed{m_2 = -5}$$

perpendicular slopes $-5 \perp \frac{1}{5} = m$

③ equation? equation: $y - y_1 = m(x - x_1)$

$$y - 2 = \frac{1}{5}(x + 1)$$

$\times 5$ $$5y - 10 = x + 1$$

Standard form $$\boxed{x - 5y + 11 = 0}$$

18. Find in terms of k the coordinates of the points where the line $3x + 4y = k$ cuts the x-axis and y-axis.
If the area of the triangle formed by $3x + 4y = k$ and the positive x and y axes is 24 square units, find the value of k.

$3x + 4y = k$

$x = 0, \; y = ?$

$3(0) + 4y = k$

$\Rightarrow y = \frac{k}{4}$

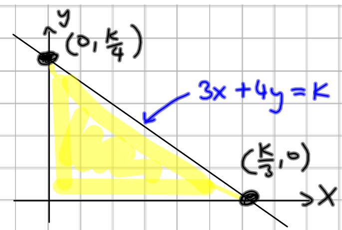
$\boxed{\text{pt } (0, k/4)}$

$y = 0, \; x = ?$

$3x + 4(0) = k$

$\Rightarrow x = \frac{k}{3}$

$\boxed{\text{pt } (\frac{k}{3}, 0)}$



$$\boxed{\text{Area} = \frac{Bh}{2}}$$

$$24 = \frac{\left(\frac{k}{3}\right)\left(\frac{k}{4}\right)}{2} \Rightarrow 48 = \frac{k^2}{12}$$

$$\Rightarrow k^2 = 576$$

$$k = 24$$