Coordinate Geometry: The Line

chapter 1

Section 1.1 Revision of formulae

PROJECT MATHS – STRAND 2
Text & Tests 4
LEAVING CERTIFICATE
HIGHER LEVEL

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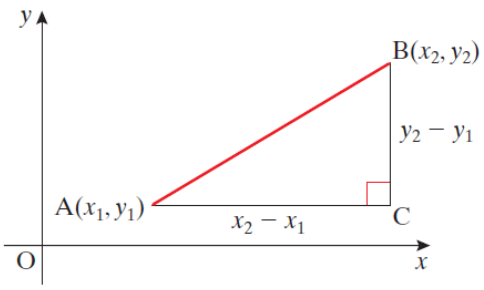
Distance between two points

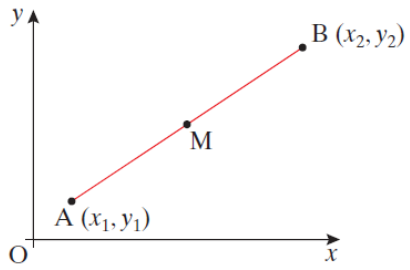
The distance between $A(x_1, y_1)$ and $B(x_2, y_2)$ is
$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



2. The midpoint of a line segment

The midpoint M of the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$\left(\frac{x_1 + x_2}{2}, \quad \frac{y_1 + y_2}{2}\right)$$



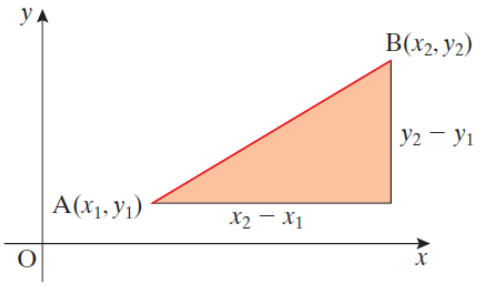
3. The slope of a line

In the diagram on the right, the slope, m, of AB is found by getting the value of

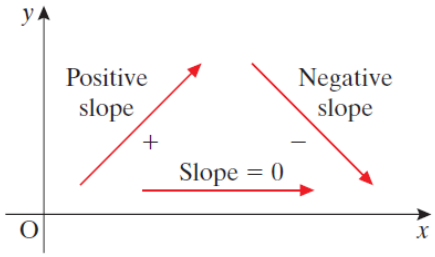
$$\frac{\text{vertical change}}{\text{horizontal change}} = \frac{y_2 - y_1}{x_2 - x_1}$$

The slope, m, of the line passing through (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



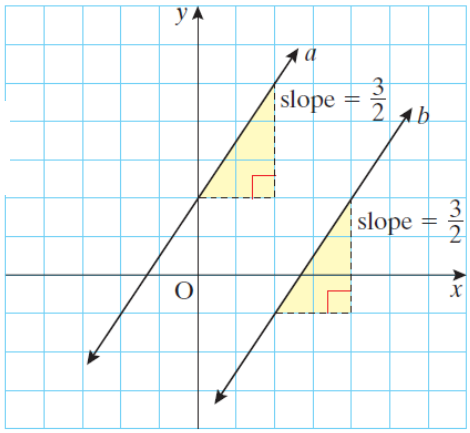
4. Positive and negative slopes



5. Parallel lines

The lines a and b in the given diagram both have the slope $\frac{3}{2}$.

Parallel lines have equal slopes.



6. Perpendicular lines

The given lines a and b are perpendicular.

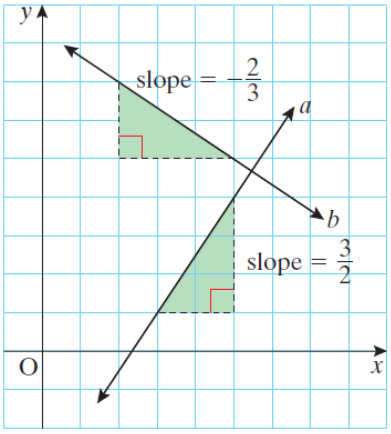
The slope of a is $\frac{3}{2}$.

The slope of $b = -\frac{2}{3}$.

Notice that one slope is the reciprocal of the other with the sign changed.

Notice also that the product of the two slopes is -1, i.e.,

$$-\frac{2}{3} \times \frac{3}{2} = -1$$



If two lines are perpendicular, the product of their slopes is -1, i.e.,

$$m_1 \times m_2 = -1$$

$\frac{2}{3} \perp -\frac{3}{2}$

Example 1

A(3, 1), B(2, −3) and C(−1, k) are three points in the plane.
If AB ⊥ AC, find the value of k.

Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$

$m_{AB} \times m_{AC} = -1$

$m_{AB} = \frac{-3-1}{2-3} = \frac{-4}{-1} = 4$

$m_{AB} = 4 \perp -\frac{1}{4} = m_{AC}$ ①

$m_{AC} = \frac{k-1}{-1-3}$

$m_{AC} = \frac{k-1}{-4}$ ②

from ① & ② →

$\frac{k-1}{-4} = -\frac{1}{4}$ $\xrightarrow{\times -4}$ $k - 1 = 1$

$\Rightarrow k = 2$

Exercise 1.1

1. Given three points A(−1, 3), B(3, −2) and C(5, 2).
Find (i) |AB| (ii) |BC| (iii) the slope of AC (iv) the midpoint of [BC].