

Chapter

5

Trigonometry 2

Section 5.4 Sum, difference and product formulae

PROJECT MATHS – STRAND 2
Text & Tests 4
 LEAVING CERTIFICATE
 HIGHER LEVEL

158

Changing products to sums or differences

$$2 \sin A \cos B = \sin(A + B) + \sin(A - B)$$

$$2 \cos A \sin B = \sin(A + B) - \sin(A - B) \quad \checkmark$$

$$2 \cos A \cos B = \cos(A + B) + \cos(A - B) \quad \checkmark$$

$$-2 \sin A \sin B = \cos(A + B) - \cos(A - B)$$

Example 1

Express as a sum or difference: (i) $2 \cos 3x \sin x$ (ii) $\cos \theta \cos 5\theta$

<p>p.15</p> <p>$A=3x$ $B=x$</p>	$2 \cos A \sin B = \sin(A+B) - \sin(A-B)$ $2 \cos 3x \sin x = \sin 4x - \sin 2x$
<p>p.15</p> <p>let $A=0$ $B=5\theta$</p>	$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$ $\Rightarrow \cos A \cos B = \frac{1}{2} [\cos(A+B) + \cos(A-B)]$ $\cos \theta \cos 5\theta = \frac{1}{2} [\cos(6\theta) + \cos(-4\theta)]$ $= \frac{1}{2} [\cos 6\theta + \cos 4\theta]$
<p>p.13 $\cos(-A) = \cos A$</p>	

Changing sums and differences to products

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

p.15

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Example 2

Express as a product (i) $\cos 5A + \cos 3A$ (ii) $\sin 3A - \sin A$

$$\begin{aligned} A &= 5A \\ B &= 3A \end{aligned}$$

$$\begin{aligned} \cos 5A + \cos 3A &= 2 \cos \left(\frac{5A+3A}{2} \right) \cos \left(\frac{5A-3A}{2} \right) \\ &= 2 \cos 4A \cos A \end{aligned}$$

Example 3

Show that $\frac{\sin 3A - \sin 2A + \sin A}{\cos 3A + \cos A - \cos 2A} = \tan 2A$.

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin 3A + \sin A = 2 \sin 2A \cos A$$

$$\cos 3A + \cos A = 2 \cos 2A \cos A$$

 \Rightarrow

$$\text{LHS} = \frac{2 \sin 2A \cos A - \sin 2A}{2 \cos 2A \cos A - \cos 2A}$$

HCF

$$= \frac{\sin 2A (2 \cos A - 1)}{\cos 2A (2 \cos A - 1)}$$

$$= \tan 2A$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$