chapter 5

Trigonometry 2

Section 5.3 Double-angle and half-angle formulae

PROJECT MATHS – STRAND 2
Text & Tests 4
LEAVING CERTIFICATE
HIGHER LEVEL

$\cos 2A = \cos^2 A - \sin^2 A$ $= 2\cos^2 A - 1$ $\cos 2A = 1 - 2\sin^2 A$	$\sin 2A = 2\sin A \cos A$ $\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$
$\cos^2 A = \frac{1}{2}(1 + \cos 2A)$	$\sin^2 A = \frac{1}{2}(1 - \cos 2A)$

Example 1

Express $\sin 3A$ in terms of $\sin A$.

$3A = 2A + A$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\cos^2 A = 1 - \sin^2 A$

$\sin 3A = \sin(2A + A)$

$= \sin 2A \cos A + \cos 2A \sin A$

$= (2\sin A\cos A)\cos A + (1 - 2\sin^2 A)\sin A$

$= 2\sin A\cos^2 A + \sin A - 2\sin^3 A$

$= 2\sin A(1 - \sin^2 A) + \sin A - 2\sin^3 A$

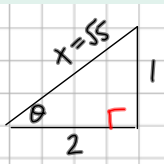
$= 2\sin A - 2\sin^3 A + \sin A - 2\sin^3 A$

$= 3\sin A - 4\sin^3 A$

$$\cos 2A = \cos^2 A - \sin^2 A \qquad \sin 2A = 2\sin A\cos A$$
$$= 2\cos^2 A - 1 \qquad \tan 2A = \frac{2\tan A}{1-\tan^2 A}$$
$$= 1 - 2\sin^2 A$$
$$\cos^2 A = \tfrac{1}{2}(1+\cos 2A) \qquad \sin^2 A = \tfrac{1}{2}(1-\cos 2A)$$

Example 2

Given that θ is acute and that $\tan\theta = \frac{1}{2}$, evaluate
(i) $\sin 2\theta$ (ii) $\cos 2\theta$.



$x^2 = 1^2 + 2^2 = 5$
$x = \sqrt{5}$

$\sin\theta = \frac{1}{\sqrt{5}}$ $\cos\theta = \frac{2}{\sqrt{5}}$

$\sin 2A = 2\sin A\cos A$ $\Rightarrow \sin 2\theta = 2\left(\frac{1}{\sqrt{5}}\right)\left(\frac{2}{\sqrt{5}}\right) = 4/5$

$\cos 2A = 1 - 2\sin^2 A$ $\Rightarrow \cos 2\theta = 1 - 2\left(\frac{1}{\sqrt{5}}\right)^2 = 1 - 2\left(\frac{1}{5}\right)$

$= \frac{3}{5}$

$$\cos 2A = \cos^2 A - \sin^2 A \qquad \sin 2A = 2\sin A\cos A$$
$$= 2\cos^2 A - 1 \qquad \tan 2A = \frac{2\tan A}{1-\tan^2 A}$$
$$= 1 - 2\sin^2 A$$
$$\cos^2 A = \tfrac{1}{2}(1+\cos 2A) \qquad \sin^2 A = \tfrac{1}{2}(1-\cos 2A)$$

Example 3

Show that (i) $\frac{\sin 2A}{1+\cos 2A} = \tan A$ (ii) $\cos^4\theta - \sin^4\theta = \cos 2\theta$.

$\sin 2A = 2\sin A\cos A$

$\cos 2A = 2\cos^2 A - 1$

LHS $= \frac{2\sin A\cos A}{1 + 2\cos^2 A - 1}$

$= \tan A$

Example 5

If $\cos 2\theta = \frac{7}{25}$, find the values of $\sin\theta$ for $0° \leqslant \theta \leqslant 360°$.

$\cos 2A = \cos^2 A - \sin^2 A$

$\cos^2 A = 1 - \sin^2 A$

$\Rightarrow \cos 2\theta = 1 - 2\sin^2 A$

$\frac{7}{25} = 1 - 2\sin^2\theta$

$-\frac{18}{25} = -2\sin^2\theta$

$\frac{9}{25} = \sin^2\theta$

$\pm\frac{3}{5} = \sin\theta$