

chapter

5

## Trigonometry 2

## Section 5.2 Compound angles

PROJECT MATHS – STRAND 2  
**Text & Tests** 4  
 LEAVING CERTIFICATE  
 HIGHER LEVEL

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Compound angle formulae

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}; \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

## Example 1

Express in surd form

(i)  $\sin 15^\circ$ (ii)  $\tan 105^\circ$ 

$$60^\circ - 45^\circ = 15^\circ$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

tables p.13

$$\begin{aligned} \sin 15^\circ &= \sin(60^\circ - 45^\circ) \\ &= \sin 60^\circ \cos 45^\circ - \cos 60^\circ \sin 45^\circ \\ &= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) \\ &= \frac{\sqrt{3} - 1}{2\sqrt{2}} \end{aligned}$$

	$30^\circ$	$60^\circ$	$45^\circ$
$\sin A$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$
$\cos A$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$
$\tan A$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	1

Compound angle formulae

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}; \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

**Example 1**

Express in surd form (i)  $\sin 15^\circ$  (ii)  $\tan 105^\circ$

$60^\circ - 45^\circ = 15^\circ$

$\sin(A - B) = \sin A \cos B - \cos A \sin B$

tables p.13

	$30^\circ$	$60^\circ$	$45^\circ$
$\sin A$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$
$\cos A$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$
$\tan A$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	1

$$\begin{aligned} \sin 15^\circ &= \sin(60^\circ - 45^\circ) \\ &= \sin 60^\circ \cos 45^\circ - \cos 60^\circ \sin 45^\circ \\ &= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) \\ &= \frac{\sqrt{3} - 1}{2\sqrt{2}} \end{aligned}$$

Compound angle formulae

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

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$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}; \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

**Example 1**

Express in surd form (i)  $\sin 15^\circ$  (ii)  $\tan 105^\circ$

$105^\circ = 60^\circ + 45^\circ$

tables p.13

	$30^\circ$	$60^\circ$	$45^\circ$
$\sin A$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$
$\cos A$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$
$\tan A$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	1

$$\begin{aligned} \tan 105^\circ &= \tan(60^\circ + 45^\circ) \\ &= \frac{\tan 60^\circ + \tan 45^\circ}{1 - \tan 60^\circ \tan 45^\circ} \\ &= \frac{\sqrt{3} + 1}{1 - (\sqrt{3} \times 1)} = \frac{\sqrt{3} + 1}{1 - \sqrt{3}} \end{aligned}$$

## Compound angle formulae

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \quad \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

## Example 2

If  $\tan A = \frac{1}{4}$  and  $\tan B = \frac{3}{5}$ , find the value of  $(A + B)$  without using a calculator.

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A+B) = \frac{\left(\frac{1}{4}\right) + \left(\frac{3}{5}\right)}{1 - \left(\frac{1}{4}\right)\left(\frac{3}{5}\right)}$$

$$\tan(A+B) = 1$$

$$\tan 45^\circ = 1$$

$$(A+B) = \tan^{-1}(1) = 45^\circ$$