chapter 5 **Trigonometry 2**

Section 5.1 Trigonometric identities

PROJECT MATHS – STRAND 2
Text & Tests 4
LEAVING CERTIFICATE HIGHER LEVEL

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1. $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$	**2.** $\sec\theta = \frac{1}{\cos\theta}$	**3.** $\tan\theta = \frac{\sin\theta}{\cos\theta}$
4. $\cot\theta = \frac{\cos\theta}{\sin\theta}$	**5.** $\sin^2\theta + \cos^2\theta = 1$	**6.** $1 + \tan^2\theta = \sec^2\theta$

Example 1

Prove these identities:

(i) $\sec A - \tan A \sin A = \cos A$ (ii) $\tan\theta\sqrt{1-\sin^2\theta} = \sin\theta$.

$\sec A = \frac{1}{\cos A}$

$\tan A = \frac{\sin A}{\cos A}$

$\sin^2 A + \cos^2 A = 1$

$\Rightarrow 1 - \sin^2 A = \cos^2 A$

$\sec A - \tan A \sin A$

$= \frac{1}{\cos A} - \frac{\sin A}{\cos A} \cdot \sin A$

$= \frac{1 - \sin^2 A}{\cos A}$

$= \frac{\cos^2 A}{\cos A}$

$= \cos A$ QED

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Example 1

Prove these identities:

(i) $\sec A - \tan A \sin A = \cos A$ (ii) $\tan\theta\sqrt{1-\sin^2\theta} = \sin\theta$.

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow 1 - \sin^2\theta = \cos^2\theta$

$\tan\theta\sqrt{1-\sin^2\theta}$

$= \dfrac{\sin\theta}{\cos\theta}\sqrt{\cos^2\theta}$

$= \dfrac{\sin\theta}{\cancel{\cos\theta}}\cancel{\cos\theta}$

$= \sin\theta$ QED

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Example 2

Prove that $\dfrac{\tan\theta + \sin\theta}{\sec\theta + 1} = \sin\theta$.

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\sec\theta = \dfrac{1}{\cos\theta}$

$LHS = \dfrac{\dfrac{\sin\theta}{\cos\theta} + \sin\theta}{\dfrac{1}{\cos\theta} + 1}$

$= \dfrac{\left(\dfrac{\sin\theta + \sin\theta\cos\theta}{\cos\theta}\right)}{\left(\dfrac{1+\cos\theta}{\cos\theta}\right)}$

$= \left(\dfrac{\sin\theta + \sin\theta\cos\theta}{\cancel{\cos\theta}}\right)\cdot\left(\dfrac{\cancel{\cos\theta}}{1+\cos\theta}\right)$

$= \sin\theta\left(\cancel{1+\cos\theta}\right)\left(\dfrac{1}{\cancel{1+\cos\theta}}\right) = \sin\theta$

Identities involving the *Sine Rule* and *Cosine Rule*

The *Sine Rule* states that $\frac{a}{\sin A} = \frac{b}{\sin B}$

This can be also written as $\sin A = \frac{a \sin B}{b}$

The *Cosine Rule* states that $\boxed{a^2 = b^2 + c^2 - 2bc \cos A}$.

This can also be written as $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$.

Example 3

Prove that in any triangle, $c \cos B - b \cos C = \frac{c^2 - b^2}{a}$

$b^2 = a^2 + c^2 - 2ac \cos B$

$c^2 = a^2 + b^2 - 2ab \cos C$

$\Rightarrow c \cos B = \frac{b^2 - a^2 - c^2}{-2a} = \frac{-b^2 + a^2 + c^2}{2a}$

$\Rightarrow -b \cos C = \frac{c^2 - a^2 - b^2}{2a}$

$\Rightarrow c \cos B - b \cos C = \frac{-b^2 + \cancel{a^2} + c^2 + c^2 - \cancel{a^2} - b^2}{2a}$

$= \frac{\cancel{2}c^2 - \cancel{2}b^2}{\cancel{2}a}$

$= \frac{c^2 - b^2}{a}$ QED

25. Use the *Sine Rule* to prove that $\frac{\sin A - \sin B}{\sin B} = \frac{a - b}{b}$.

$\frac{\sin A}{a} = \frac{\sin B}{b}$

$\frac{\sin A}{\sin B} = \frac{a}{b}$

$\frac{\sin B}{\sin B} = 1$

$LHS = \frac{\sin A}{\sin B} - \frac{\sin B}{\sin B}$

$= \frac{a}{b} - 1$

$= \frac{a-b}{b}$