Question 5 **(25 marks)**

A is the closed interval $[0,5]$. That is, $A = \{x \mid 0 \le x \le 5, x \in \mathbb{R}\}$.

The function f is defined on A by:

$$f : A \to \mathbb{R} : x \mapsto x^3 - 5x^2 + 3x + 5.$$

(a) Find the maximum and minimum values of f.

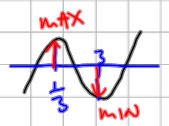
at Vertex $f'(x) = 0$

$\Rightarrow f'(x) = 3x^2 - 10x + 3 = 0$

$(3x - 1)(x - 3) = 0$

$x = \frac{1}{3}, \quad x = 3$

SHAPE :



$f(\frac{1}{3}) = (\frac{1}{3})^3 - 5(\frac{1}{3})^2 + 3(\frac{1}{3}) + 5$

$= \frac{148}{27} \approx 5.5$

$f(3) = (3)^3 - 5(3)^2 + 3(3) + 5$

$= -4$

max pt $(\frac{1}{3}, \frac{148}{27})$ high

min pt $(3, -4)$ low

FROMAL WAY OF DISTINGUISHING MAX/MIN :

at max $f''(x) < 0$

min $f''(x) > 0$

$\Rightarrow f'(x) = 3x^2 - 10x + 3$

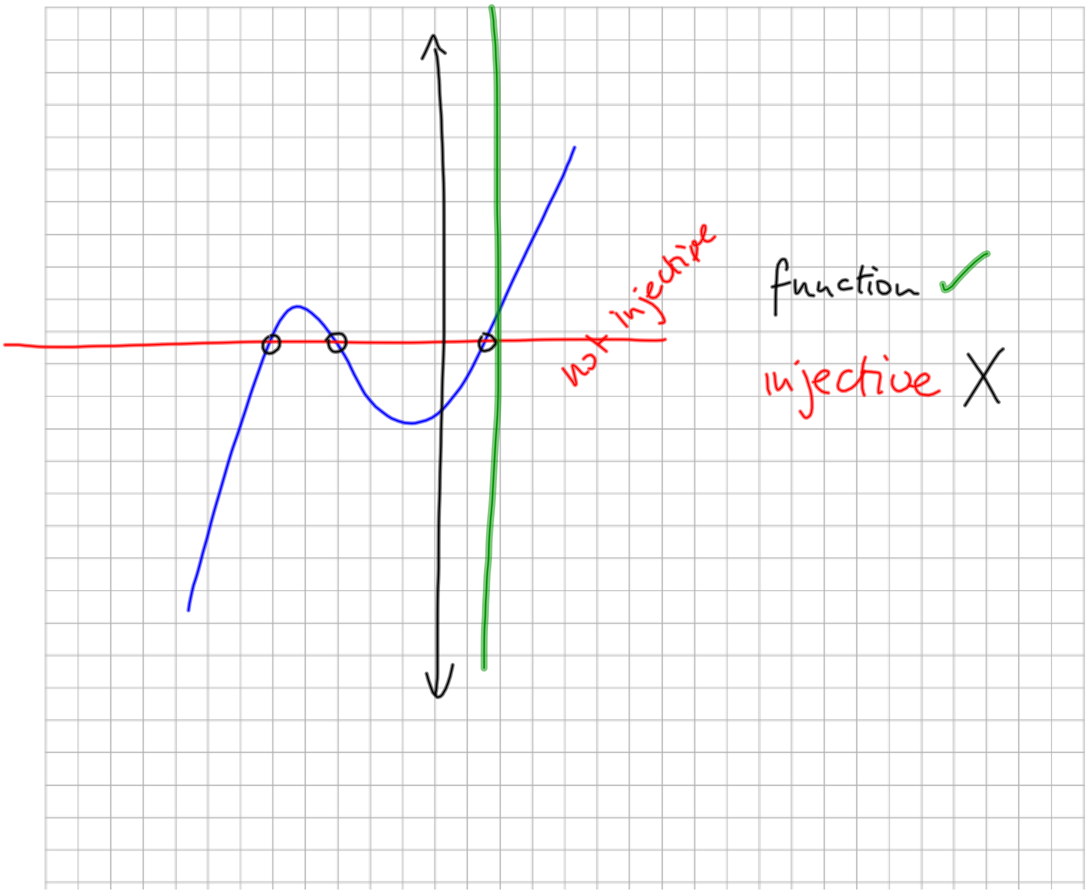
$f''(x) = 6x - 10$

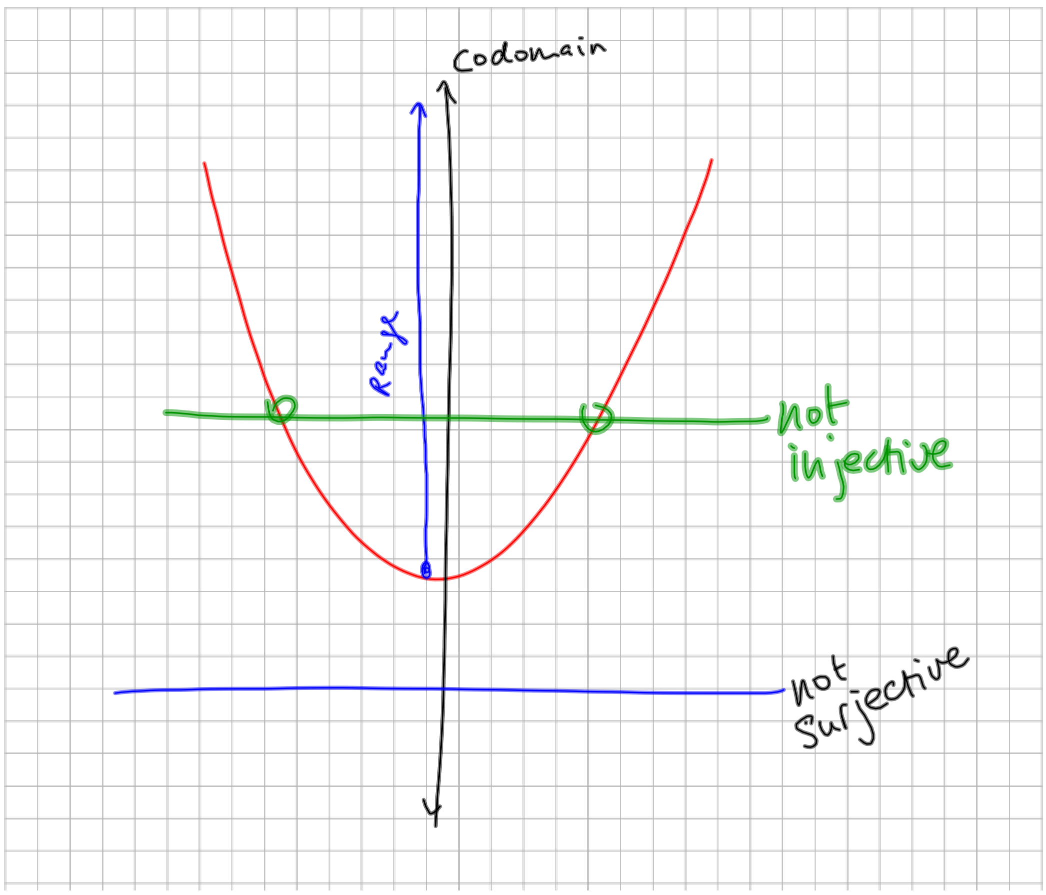
at $x = \frac{1}{3}$ $\Rightarrow f''(\frac{1}{3}) = 6(\frac{1}{3}) - 10 = -8 < 0$

$\Rightarrow$ at max $x = \frac{1}{3}$

$x = 3$ $\Rightarrow f''(3) = 6(3) - 10 = 8 > 0$

$\Rightarrow$ at min $x = 3$


not injective
function ✓
injective X


Codomain
Range
not injective
not surjective

(b) The graphs of the functions $f: x \mapsto |x-3|$ and $g: x \mapsto 2$ are shown in the diagram.

(i) Find the co-ordinates of the points A, B, C and D.

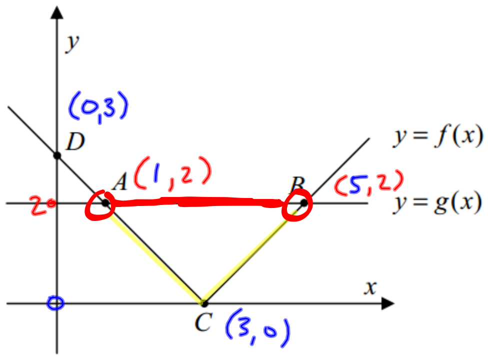
$f(0) = |0-3| = 3$ pt $(0,3)$

If $|x-3| = 0 \Rightarrow x = 3$ pt $(3,0)$

If $|x-3| = 2 \Rightarrow$ either $x-3=2 \Rightarrow x=5$ pt $(5,2)$ or $x-3=-2 \Rightarrow x=1$ pt $(1,2)$

$A = (1, 2)$ $B = (5, 2)$

$C = (3, 0)$ $D = (0, 3)$



(ii) Hence, or otherwise, solve the inequality $|x-3| < 2$.

from graph $1 < x < 5$