Work done this year

- Algebra (* not all Algebra 3)
- Complex Numbers
- Sequences..
- Financial maths
- Area, Volume
- Probability

Remaining classes

Plan

M	T	W	T	F
	6	7 METHEL	8	9
12 BIOLOGY	13 TRIPS	14 HOLIDAYS	~~15~~ SPORTS	16 GEOGRAPHY
19	20	21 LOURDES	22	23 *ID
~~26~~ EXAMS	~~27~~	~~28~~	~~29~~	

Complex Number Revision

Express $z = \dfrac{\sqrt{3}+i}{1+i\sqrt{3}}$ in polar form. Hence evaluate $\left(\dfrac{\sqrt{3}+i}{1+i\sqrt{3}}\right)^6$

- Divide

multiply above and below by conjugate of denominator

$$z = \frac{(\sqrt{3}+i)(1-i\sqrt{3})}{(1+i\sqrt{3})(1-i\sqrt{3})} = \frac{\sqrt{3}-i3+i\cancel{-}{+}i^2\sqrt{3}}{1+3} = \frac{2\sqrt{3}-2i}{4}$$

$$z = \frac{\sqrt{3}}{2} - \frac{1}{2}i$$

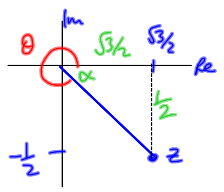
- Change to Polar form

$r = ?$ $\theta = ?$

$r = \sqrt{a^2+b^2}$

$$r = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{3}{4}+\frac{1}{4}} = \sqrt{1} = 1$$

Sketch



$\alpha = \tan^{-1}\left(\dfrac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) = 30°$

$30° \Leftrightarrow \frac{\pi}{6}$ Radians

$\Rightarrow \theta = 2\pi - \frac{\pi}{6} = \frac{11}{6}\pi$

Polar form

$$\Rightarrow z = 1\,\mathrm{Cis}\,\tfrac{11}{6}\pi = \mathrm{Cis}\,\tfrac{11}{6}\pi$$

- Use de Moivre's theorem

$[r\,\mathrm{Cis}\,\theta]^n = r^n\,\mathrm{Cis}\,n\theta$

$$z^6 = \mathrm{Cis}\,\cancel{6}\left(\tfrac{11}{\cancel{6}}\pi\right) = \cos 11\pi + i\sin 11\pi$$

evaluate with calculator

$$z^6 = -1 + i0$$

$$\Rightarrow z^6 = -1$$

Section 3.10

5. Let z be the complex number $-1 + i\sqrt{3}$.

(i) Express z^2 in the form $a + bi$.

(ii) Find the value of the real number p such that $z^2 + pz$ is real.

(i) $z^2 = (-1+i\sqrt{3})^2$ $\quad (a+b)^2 = a^2+2ab+b^2$

$= 1 - 2i\sqrt{3} \cancel{+} \cancel{i^2}3$

$\boxed{z^2 = -2 - 2\sqrt{3}\,i}$

(ii) $z^2 + pz = \text{Real}$ $\quad (\Rightarrow \text{Imaginary part} = 0)$

$\Rightarrow -2-2\sqrt{3}\,i + p(-1+i\sqrt{3}) = \text{Real}$

$-2-2\sqrt{3}\,i - p + p\sqrt{3}\,i = \text{Real}$

$\underbrace{(-2-p)}_{Re} + \underbrace{(p\sqrt{3}-2\sqrt{3})i}_{Im} = \text{Real}$

$\Rightarrow p\sqrt{3} - 2\sqrt{3} = 0$

$\Rightarrow \boxed{p=2}$

Section 3.10

9. If $z_1 = 2+3i$ and $z_2 = 1-4i$, investigate if $|z_1|.|z_2| = |z_1.z_2|$

$|z_1| = \sqrt{2^2+3^2} = \sqrt{4+9} = \sqrt{13}$

$|z_2| = \sqrt{1^2+4^2} = \sqrt{1+16} = \sqrt{17}$

$|z_1|.|z_2| = \sqrt{13}\sqrt{17} = \boxed{\sqrt{221}}$

$z_1.z_2 = (2+3i)(1-4i)$

$= 2-8i+3i \cancel{-}+12\cancel{i^2}$

$= 14-5i$

$|z_1.z_2| = \sqrt{14^2+5^2} = \sqrt{196+25} = \boxed{\sqrt{221}}$

$\Rightarrow |z_1|.|z_2| = |z_1.z_2|$