Complex Number Revision

T&T6 Chapter 3

1. Simplify $\sqrt{80} - \sqrt{20}$, expressing your answer in the form $a\sqrt{b}$ where $a, b \in \mathbb{N}$.

$= \sqrt{16 \times 5} - \sqrt{4 \times 5}$

$= 4\sqrt{5} - 2\sqrt{5}$

$= 2\sqrt{5}$

2. $\underbrace{(x-1)+yi}_{LHS} = \underbrace{y+4i}_{RHS}$; find x and y.

Re = Re
Im = Im

$x - 1 = y$
$y = 4$ ✓

$\Rightarrow x - 1 = 4$
$\Rightarrow x = 5$ ✓

3. Solve the equation $z^2 + 4z + 3 = 0$, giving your answer in the form $a + bi$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1$
$b = 4$
$c = 3$

4. Given $z_1 = 5 + i$ and $z_2 = -2 + 3i$.

(i) Find $(z_1)^2$ (ii) Show that $|z_1|^2 = 2|z_2|^2$

(i)

$(a+b)^2 = a^2 + 2ab + b^2$

$z_1^2 = (5+i)^2 = 25 + 10i + \cancel{i^2}$

$= 24 + 10i$

(ii)

modulus

$|a+bi| = \sqrt{a^2+b^2}$

$|z_1| = |5+i| = \sqrt{5^2+1^2} = \sqrt{26}$

$|z_1|^2 = (\sqrt{26})^2 = 26$

$|z_2| = |-2+3i| = \sqrt{2^2+3^2} = \sqrt{13}$

$2|z_2|^2 = 2(\sqrt{13})^2 = 2(13) = 26$

$\Rightarrow |z_1|^2 = 2|z_2|^2$

5. Let z be the complex number $-1 + \sqrt{3}i$.

(i) Express z^2 in the form $a + bi$.

(ii) Find the value of the real number p such that $z^2 + pz$ is real.

(i)

$(a+b)^2 = a^2 + 2ab + b^2$

$z^2 = (-1+\sqrt{3}i)^2$

$= 1 - 2\sqrt{3}i + 3\cancel{i^2}$

$= -2 - 2\sqrt{3}i$

(ii) $z^2 + pz = \text{Real}$

$z^2 + pz = -2 - 2\sqrt{3}i + p(-2 - 2\sqrt{3}i)$

$= -2 - 2\sqrt{3}i - 2p - 2p\sqrt{3}i$

$= \underbrace{(-2-2p)}_{\text{Real}} - \underbrace{(2\sqrt{3} + 2p\sqrt{3})i}_{\text{imaginary}}$

i part = 0i

$\Rightarrow 2\sqrt{3} + 2p\sqrt{3} = 0$

$2\sqrt{3}p = -2\sqrt{3}$

$p = -\frac{2\sqrt{3}}{2\sqrt{3}} \Rightarrow \boxed{p = -1}$