Chapter 1: Algebra 1

Exercise 1.1

Q**1**. (i) coefficient of $x^2 = 3$

(ii) coefficient of $x = -9$

(iii) independent term = 5.

Q**2**. (i) degree 2

(ii) degree 3

(iii) degree 4.

Q**3**. $\frac{-4}{x} = -4x^{-1}$, -1 is not a positive power;

$x^{3/2}$, $3/2$ is not an integer.

Q**4**. (i) $3x^2 - 6x + 7 + 5x^2 + 2x - 9 = 8x^2 - 4x - 2$

(ii) $x^3 - 4x^2 - 5x + 3x^3 + 6x^2 - x = 4x^3 + 2x^2 - 6x$

(iii) $x(x+4) + 3x(2x-3) = x^2 + 4x + 6x^2 - 9x$

$= 7x^2 - 5x$

(iv) $3(x^2 - 7) + 2x(3x-1) - 7x + 2 = 3x^2 - 21 + 6x^2 - 2x - 7x + 2$

$= 9x^2 - 9x - 19$

Q**5**. (i) $3x^2(4x+2) + 5x^2(2x-5) = 12x^3 + 6x^2 + 10x^3 - 25x^2$

$= 22x^3 - 19x^2$

(ii) $x^3(x-2) + 4x^3(2x-6) = x^4 - 2x^3 + 8x^4 - 24x^3$

$= 9x^4 - 26x^3$

(iii) $x(x^3 + 4x^2 - 7x) + 3x^2(2x^2 - 3x + 4) = x^4 + 4x^3 - 7x^2 + 6x^4 - 9x^3 + 12x^2$

$= 7x^4 - 5x^3 + 5x^2$

(iv) $3x(x^2 - 7x + 1) + 2x^2(6x - 5) = 3x^3 - 21x^2 + 3x + 12x^3 - 10x^2$

$= 15x^3 - 31x^2 + 3x$

Q**6**. (i) $(x+4)(2x+5) = 2x^2 + 5x + 8x + 20 = 2x^2 + 13x + 20$

(ii) $(2x+3)(x-2) = 2x^2 - 4x + 3x - 6 = 2x^2 - x - 6$

(iii) $(3x-2)(x+3) = 3x^2 + 9x - 2x - 6 = 3x^2 + 7x - 6$

(iv) $(3x-2)(4x-1) = 12x^2 - 3x - 8x + 2 = 12x^2 - 11x + 2$

(v) $(3x-1)(2x+5) = 6x^2 + 15x - 2x - 5 = 6x^2 + 13x - 5$

(vi) $(4x+1)(2x-6) = 8x^2 - 24x + 2x - 6 = 8x^2 - 22x - 6$

(vii) $(x-2)(x+2)=x^2+2x-2x-4=x^2-4$

(viii) $(2x+5)(2x-5)=4x^2-10x+10x-25=4x^2-25$

(ix) $(ax-by)(ax+by)=a^2x^2+abxy-abxy-b^2y^2=a^2x^2-b^2y^2$

Q**7**. (i) $(x+2)^2=x^2+4x+4$

(ii) $(x-3)^2=x^2-6x+9$

(iii) $(x+5)^2=x^2+10x+25$

(iv) $(a+b)^2=a^2+2ab+b^2$

(v) $(x-y)^2=x^2-2xy+y^2$

(vi) $(a+2b)^2=a^2+4ab+4b^2$

(vii) $(3x-y)^2=9x^2-6xy+y^2$

(viii) $(x-5y)^2=x^2-10xy+25y^2$

(ix) $(2x+3y)^2=4x^2+12xy+9y^2$

Q**8**. (i) $\left(x+\frac{1}{2}\right)^2=x^2+2(x)\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2=x^2+x+\frac{1}{4}$

(ii) $8\left(x-\frac{1}{4}\right)^2=8\left(x^2-2(x)\left(\frac{1}{4}\right)+\left(\frac{-1}{4}\right)^2\right)=8\left(x^2-\frac{x}{2}+\frac{1}{16}\right)$

$$=8x^2-4x+\frac{1}{2}$$

(iii) $-(1-x)^2=-(1-2x+x^2)=-1+2x-x^2$

Q**9**. (i) $x^2+5x+25$; No, cannot be written in the form $(x+k)^2$

(ii) $9x^2-6x-1$; No, " " " " " " "

(iii) $4+12x+9x^2=(2+3x)^2$; YES.

Q**10**. $px^2+4x+1=(ax+1)^2$

px^2+4x+1 can be written in the form $(ax+1)^2=a^2x^2+2ax+1$

$\therefore 2a=4\Rightarrow a=2$

$\therefore (ax+1)^2=(2x+1)^2=4x^2+4x+1$

$\therefore p=4.$

Q**11**. $25x^2+tx+4=(5x+2)^2$ as a perfect square

$=25x^2+20x+4$

$\Rightarrow t=20.$

Q**12**. $9x^2 + 24x + s = (3x+a)^2 = 9x^2 + 6ax + a^2$

$$\Rightarrow 6a = 24$$

$$a = 4.$$

$$\therefore 9x^2 + 24x + s = (3x+4)^2$$

$$= 9x^2 + 24x + 16$$

$$\Rightarrow s = 16.$$

Q**13**. (i) $(x+2)(x^2+2x+6) = x^3 + 2x^2 + 6x + 2x^2 + 4x + 12$

$$= x^3 + 4x^2 + 10x + 12$$

(ii) $(x-4)(2x^2+3x-1) = 2x^3 + 3x^2 - x - 8x^2 - 12x + 4$

$$= 2x^3 - 5x^2 - 13x + 4$$

(iii) $(2x+3)(x^2 - 3x + 2) = 2x^3 - 6x^2 + 4x + 3x^2 - 9x + 6$

$$= 2x^3 - 3x^2 - 5x + 6$$

(iv) $(3x-2)(2x^2 - 4x + 2) = 6x^3 - 12x^2 + 6x - 4x^2 + 8x - 4$

$$= 6x^3 - 16x^2 + 14x - 4.$$

Q**14**. $(x+y)(x^2 - xy + y^2) = x^3 - \cancel{x^2y} + \cancel{xy^2} + \cancel{x^2y} - \cancel{xy^2} + y^3$

$$= x^3 + y^3$$

Q**15**. $(x-y)(x^2 + xy + y^2) = x^3 + \cancel{x^2y} + \cancel{xy^2} - \cancel{x^2y} - \cancel{xy^2} - y^3$

$$= x^3 - y^3$$

Q**16**. $(2x-3)(3x^2 - 2x + 4) = 6x^3 - 4x^2 + 8x - 9x^2 + 6x - 12$

$$= 6x^3 - 13x^2 + 14x - 12$$

$\Rightarrow$ coefficient of $x = 14$.

Q**17**. $(x+3)(x-4)(2x+1) = (x+3)(2x^2 + x - 8x - 4)$

$$= (x+3)(2x^2 - 7x - 4)$$

$$= 2x^3 - 7x^2 - 4x + 6x^2 - 21x - 12$$

$$= 2x^3 - x^2 - 25x - 12.$$

Q**18**. $(x^2 - 3x - 2)(2x^2 - 4x + 1) = 2x^4 - 4x^3 + x^2 - 6x^3 + 12x^2 - 3x$

$$- 4x^2 + 8x - 2$$

$$= 2x^4 - 10x^3 + 9x^2 + 5x - 2.$$

Q**19**. $(3x^2 + 5x - 1)(2x^2 - 6x - 5);$ x^2 coefficients include $-15x^2 - 30x^2 - 2x^2$

$$= -47x^2$$

coefficient of $x^2 = -47.$

Q20. (i) $\dfrac{3x+6}{3} = x+2$

(ii) $\dfrac{x^2+2x}{x} = x+2$

(iii) $\dfrac{3x^3-6x^2}{3x} = \dfrac{3x^2(x-2)}{3x} = x(x-2) = x^2-2x$

(iv) $\dfrac{15x^2y-10xy^2}{5xy} = 3x-2y$

Q21. (i) $\dfrac{6x^2y+9xy^2-3xy}{3xy} = x+3y-1$

(ii) $\dfrac{6x^4-9x^3+12x^2}{3x^2} = \dfrac{3x^2(x^2-3x+4)}{3x^2}$

$= x^2-3x+4$

Q22. (i) $\dfrac{\overset{4}{\cancel{12}}\,a^{\cancel{2}}\cancel{b}}{\cancel{3}\cancel{a}\cancel{b}} = 4a$

(ii) $\dfrac{\overset{4}{\cancel{12}}\,a^{\cancel{2}}b\cancel{c}}{\cancel{3}\cancel{a}\cancel{c}} = 4ab$

(iii) $\dfrac{\overset{2}{\cancel{4}}\,\cancel{x}y^{\cancel{2}}z}{\cancel{2}\cancel{x}\cancel{y}} = 2yz$

(iv) $\dfrac{3xy}{2}\cdot\dfrac{4}{6x^2} = \dfrac{\cancel{12}\,\cancel{x}y}{\cancel{12}\,x^{\cancel{2}}} = \dfrac{y}{x}$

Q23. (i) $\dfrac{2x^2+5x-3}{2x-1} = \dfrac{\cancel{(2x-1)}\,(x+3)}{\cancel{2x-1}} = x+3$

(ii) $\dfrac{2x^2-2x-12}{x-3} = \dfrac{2(x^2-x-6)}{x-3} = \dfrac{2\,\cancel{(x-3)}\,(x+2)}{\cancel{x-3}}$

$= 2(x+2) = 2x+4$

(iii) $\dfrac{8x^2+8x-6}{4x-2} = \dfrac{2(4x^2+4x-3)}{2(2x-1)} = \dfrac{\cancel{2}\,\cancel{(2x-1)}\,(2x+3)}{\cancel{2}\,\cancel{(2x-1)}}$

$= 2x+3$

Q**24**. (i)

$$\begin{array}{r|l}
 & x^2-7x+12 \\ \hline
x-1 & x^3-8x^2+19x-12 \\
 & \underline{x^3-x^2} \\
 & \quad -7x^2+19x-12 \\
 & \quad \underline{-7x^2+7x} \\
 & \qquad\qquad 12x-12 \\
 & \qquad\qquad \underline{12x-12}
\end{array}$$

(ii)

$$\begin{array}{r|l}
 & x^2-1 \\ \hline
2x-1 & 2x^3-x^2-2x+1 \\
 & \underline{2x^3-x^2} \\
 & \qquad\qquad -2x+1 \\
 & \qquad\qquad \underline{-2x+1}
\end{array}$$

(iii)

$$\begin{array}{r|l}
 & x^2-1 \\ \hline
3x-4 & 3x^3-4x^2-3x+4 \\
 & \underline{3x^3-4x^2} \\
 & \qquad\qquad -3x+4 \\
 & \qquad\qquad \underline{-3x+4}
\end{array}$$

(iv)

$$\begin{array}{r|l}
 & 4x^2+5x-6 \\ \hline
x-3 & 4x^3-7x^2-21x+18 \\
 & \underline{4x^3-12x^2} \\
 & \quad +5x^2-21x+18 \\
 & \quad \underline{+5x^2-15x} \\
 & \qquad\qquad -6x+18 \\
 & \qquad\qquad \underline{-6x+18}
\end{array}$$

(v)

$$\begin{array}{r|l}
 & x^2-5x+3 \\ \hline
x+5 & x^3 \qquad -22x+15 \\
 & \underline{x^3+5x^2} \\
 & \quad -5x^2-22x+15 \\
 & \quad \underline{-5x^2-25x} \\
 & \qquad\qquad +3x+15 \\
 & \qquad\qquad \underline{+3x+15}
\end{array}$$

(vi)

$$\begin{array}{r|l}
 & 2x^2+3x+6 \\ \hline
x-2 & 2x^3-x^2 \qquad -12 \\
 & \underline{2x^3-4x^2} \\
 & \qquad 3x^2 \qquad -12 \\
 & \qquad \underline{3x^2-6x} \\
 & \qquad\qquad 6x-12 \\
 & \qquad\qquad \underline{6x-12}
\end{array}$$

Q25. (i)

$$\begin{array}{r|l} & x-2 \\ \hline x^2+2 & x^3-2x^2+2x-4 \\ & \underline{x^3 \qquad\quad +2x} \\ & \quad -2x^2 \qquad -4 \\ & \quad \underline{-2x^2 \qquad -4} \end{array}$$

(ii)

$$\begin{array}{r|l} & x-3 \\ \hline x^2-6x+9 & x^3-9x^2+27x-27 \\ & \underline{x^3-6x^2+9x} \\ & \qquad -3x^2+18x-27 \\ & \qquad \underline{-3x^2+18x-27} \end{array}$$

(iii)

$$\begin{array}{r|l} & 3x-1 \\ \hline x^2+x-2 & 3x^3+2x^2-7x+2 \\ & \underline{3x^3+3x^2-6x} \\ & \qquad -x^2-x+2 \\ & \qquad \underline{-x^2-x+2} \end{array}$$

(iv)

$$\begin{array}{r|l} & x+2 \\ \hline 5x^2+4x-1 & 5x^3+14x^2+7x-2 \\ & \underline{5x^3+4x^2-x} \\ & \qquad +10x^2+8x-2 \\ & \qquad \underline{+10x^2+8x-2} \end{array}$$

Q26. (i)

$$\begin{array}{r|l} & x^2+2x+4 \\ \hline x-2 & x^3 \qquad\qquad -8 \\ & \underline{x^3-2x^2} \\ & \qquad 2x^2 \qquad -8 \\ & \qquad \underline{2x^2-4x} \\ & \qquad\qquad 4x-8 \\ & \qquad\qquad 4x-8 \end{array}$$

(ii)

$$\begin{array}{r|l} & 4x^2+6xy+9y^2 \\ \hline 2x-3y & 8x^3 \qquad\qquad\qquad -27y^3 \\ & \underline{8x^3-12x^2y} \\ & \qquad 12x^2y \qquad\quad -27y^3 \\ & \qquad \underline{12x^2y-18xy^2} \\ & \qquad\qquad\quad 18xy^2-27y^3 \\ & \qquad\qquad\quad 18xy^2-27y^3 \end{array}$$

Exercise 1.2

Q**1**. x cm length of smaller side,

$(x + 4)$ cm length of longer side.

(i) $A(x) = x(x+4) = (x^2 + 4x)\text{ cm}^2$

(ii) $P(x) = 2[x + (x+4)] = (4x+8)\text{ cm}$

Q**2**. (i) $\text{Area} = \text{length} \times \text{width}$

$$\Rightarrow \text{width} = \frac{\text{Area}}{\text{length}} = \frac{6x^2 + 4x - 2}{3x-1}$$

$$= \frac{(3x-1)(2x+2)}{3x-1} = 2x+2$$

(ii) $\text{Perimeter} = 2(\text{length} + \text{width})$

$$= 2((3x-1) + (2x+2))$$

$$P(x) = 10x + 2$$

Q**3**. (a) $V(x) = (2x+3)(x)(x+1)$

$$= (2x+3)(x^2 + x)$$

$$= 2x^3 + 2x^2 + 3x^2 + 3x$$

$$= 2x^3 + 5x^2 + 3x$$

(b) $S(x) = (x)(2x+3) + 2(x)(x+1) + 2(2x+3)(x+1)$

$$= 2x^2 + 3x + 2x^2 + 2x + 4x^2 + 4x + 6x + 6$$

$$= 8x^2 + 15x + 6$$

(c) (i) $V(5) = 2(5)^3 + 5(5)^2 + 3(5) = 390\text{ cm}^3$

(ii) $S(5) = 8(5)^2 + 15(5) + 6 = 281\text{ cm}^2$

Q**4**. $f(x) = 2x^3 - x^2 - 5x - 4$

(a) $f(0) = 2(0)^3 - (0)^2 - 5(0) - 4 = -4$

(b) $f(1) = 2(1)^3 - (1)^2 - 5(1) - 4 = -8$

(c) $f(-2) = 2(-2)^3 - (-2)^2 - 5(-2) - 4 = -14$

(d) $f(3a) = 2(3a)^3 - (3a)^2 - 5(3a) - 4 = 54a^3 - 9a^2 - 15a - 4$

Q**5**. $f(x) = x^2 - 3x + 6$

(a) $f(0) = (0)^2 - 3(0) + 6 = 6$

(b) $f(-5) = (-5)^2 - 3(-5) + 6 = 46$

(c) $f\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^2 - 3\left(-\frac{1}{2}\right) + 6 = \frac{31}{4} = 7.75$

(d) $f\left(\frac{a}{4}\right) = \left(\frac{a}{4}\right)^2 - 3\left(\frac{a}{4}\right) + 6 = \frac{a^2}{16} - \frac{3a}{4} + 6$

Q**6**. Length $= (x - y)$.

Width $= (2x + 3y)$.

(a) Area $= (x-y)(2x+3y) = 2x^2 + 3xy - 2xy - 3y^2$

$$= 2x^2 + xy - 3y^2$$

(b) Perimeter $= 2[(x-y)+(2x+3y)] = 2[3x+2y] = 6x+4y$

Q**7**. Length $= x$ cm

Width $= (x-5)$ cm

Height $= 2x$ cm.

(a) Volume $=$ Length $\times$ Width $\times$ Height

$$= (x)(x-5)(2x) = 2x^3 - 10x^2$$

(b) Surface area $= 2(x)(x-5) + 4(2x)(x-5) + 4(2x)(x)$

$$= 2x^2 - 10x + 8x^2 - 40x + 8x^2$$

$$= 18x^2 - 50x.$$

Q**8**. (i) $d(4)$ = the number of diagonals in a 4-sided polygon

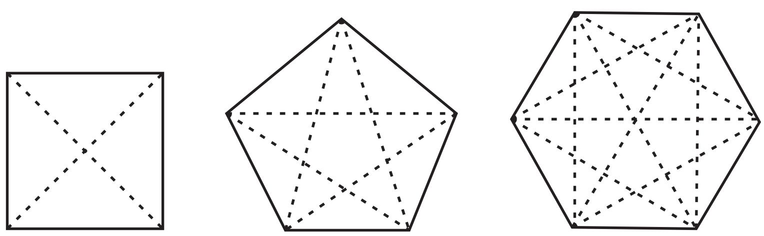
(ii) $d(5)$ = the number of diagonals in a 5-sided polygon.

$$d(4) = \frac{(4)^2}{2} - \frac{3(4)}{2} = 8 - 6 = 2.$$

$$d(5) = \frac{(5)^2}{2} - \frac{3(5)}{2} = \frac{25}{2} - \frac{15}{2} = \frac{10}{2} = 5.$$

$$d(6) = \frac{(6)^2}{2} - \frac{3(6)}{2} = \frac{36}{2} - \frac{18}{2} = \frac{18}{2} = 9.$$

$d(3) = 0$ because a triangle has no diagonal.



Q**9**. If $f(x) = x + 5$,

$$f(a^2) - 3f(a) + 2 = a^2 + 5 - 3(a+5) + 2$$

$$= a^2 + 5 - 3a - 15 + 2$$

$$= a^2 - 3a - 8.$$

Q**10**. $f(x) = x^2 - 3x + 6$.

(i) $f(-2t) = (-2t)^2 - 3(-2t) + 6 = 4t^2 + 6t + 6.$

(ii) $f(t^2) = (t^2)^2 - 3(t^2) + 6 = t^4 - 3t^2 + 6.$

(iii) $f(t-2)=(t-2)^2-3(t-2)+6=t^2-4t+4-3t+6+6.$

$$=t^2-7t+16.$$

(i) $4t^2+6t+6$ is of degree 2.

(ii) t^4-3t^2+6 is of degree 4.

(iii) $t^2-7t+16$ is of degree 2.

Q**11**. $V(r,h)=\frac{1}{3}\pi r^2 h.$

(i) $V(r=14,h=21)=\frac{1}{3}\pi\cdot 14^2\cdot 21=1372\pi\ \text{cm}^3$

(ii) $V(r,h=r)=\frac{1}{3}\pi\cdot r^2\cdot r=\frac{1}{3}\pi\, r^3$

(iii) $V(r=2h,h)=\frac{1}{3}\pi(2h)^2\cdot h=\frac{4}{3}\pi\, h^3$

Q**12**.

$$f(x)=3x+6.$$
$$f(10)=3(10)+6=36$$
$$f(x)=2x+8.$$
$$f(10)=2(10)+8=28$$
$$g(10)=47=40+7=4(10)+7$$
$$\therefore g(x)=4x+7$$

Q**13**.

$$T=2\pi\sqrt{\frac{l}{g}}$$

$$\therefore T^2=4\pi^2\,\frac{l}{g}$$

$$\therefore \frac{gT^2}{4\pi^2}=l$$

$$\therefore l=\frac{gT^2}{4\pi^2}$$

$\Rightarrow$ when $T=4$ and $g=10$, $l=\frac{10.4^2}{4\pi^2}=\frac{40}{\pi^2}$ m.

Q**14**.

$$V=\frac{4}{3}\pi r^3$$

$$\Rightarrow \frac{4}{3}\pi r^3=V$$

$$\Rightarrow r^3=\frac{3V}{4\pi}$$

$$\Rightarrow r=\sqrt[3]{\frac{3V}{4\pi}}$$

when $V = \frac{792}{7}$ and $\pi = \frac{22}{7}$, $r = \sqrt[3]{\frac{3 \times \overset{198}{\cancel{792}} \times \cancel{7}}{\cancel{4} \times \cancel{7} \times 22}} = \sqrt[3]{27}$

$= 3$ m

Q**15**. $H(x) = \frac{x}{2}(x-1)$, $x =$ number of students.

(i) $x = 5 \Rightarrow H(5) = \frac{5}{2}(5-1) = 10$

(ii) $x = 6 \Rightarrow H(6) = \frac{6}{2}(6-1) = 15$

(iii) $x = 10 \Rightarrow H(10) = \frac{10}{2}(10-1) = 45$

(iv) $H(x) = 136 = \frac{x}{2}(x-1)$

$272 = x(x-1)$

$\Rightarrow$ The product of two consecutive numbers $= 272$.

$\Rightarrow$ if $x = 16$, $x - 1 = 15$ $\therefore$ $16 \times 15 = 240$.

if $x = 17$, $x - 1 = 16$ $\therefore$ $17 \times 16 = 272$.

$\therefore$ $x = 17$.

or

$272 = x^2 - x$

$\Rightarrow x^2 - x - 272 = 0$

$(x-17)(x+16) = 0$

$\therefore$ $x - 17 = 0 \Rightarrow x = 17$

or $x + 16 = 0 \Rightarrow x = -16$ which is invalid

since x stands for the number of students

$\therefore$ $x = 17$.

Exercise 1.3

Q**1**. $5x^2 - 10x = 5x(x-2)$

Q**2**. $6ab - 12bc = 6b(a - 2c)$

Q**3**. $3x^2 - 6xy = 3x(x - 2y)$

Q**4**. $2x^2y - 6x^2z = 2x(xy - 3z)$

Q**5**. $2a^3 - 4a^2 + 8a = 2a(a^2 - 2a + 4)$

Q**6**. $5xy^2 - 20x^2y = 5xy(y - 4x)$

Q**7**. $2a^2b - 4ab^2 + 12abc = 2ab(a - 2b + 6c)$

Q**8**. $3x^2y - 9xy^2 + 15xyz = 3xy(x - 3y + 5z)$

Q**9**. $4\pi r^2 + 6\pi rh = 2\pi r(r + 3h)$

Q**10**. $3a(2b - c) - 4(2b - c) = (2b - c)(3a - 4)$

Q11. $x^2 - ax + 3x - 3a = x(x-a) + 3(x-a)$
$= (x-a)(x+3)$

Q12. $2c^2 - 4cd + c - 2d = 2c(c-2d) + c - 2d$
$= (c-2d)(2c+1)$

Q13. $8ax + 4ay - 6bx - 3by = 4a(2x+y) - 3b(2x+y)$
$= (2x+y)(4a-3b)$

Q14. $7y^2 - 21by + 2ay - 6ab = 7y(y-3b) + 2a(y-3b)$
$= (y-3b)(7y+2a)$

Q15. $6xy + 12yz - 8xz - 9y^2 = 6xy - 9y^2 + 12yz - 8xz$
$= 3y(2x-3y) + 4z(3y-2x)$
$= 3y(2x-3y) - 4z(2x-3y)$
$= (2x-3y)(3y-4z)$

Q16. $6x^2 - 3y(3x-2a) - 4ax = 6x^2 - 4ax - 3y(3x-2a)$
$= 2x(3x-2a) - 3y(3x-2a)$
$= (3x-2a)(2x-3y)$

Q17. $3ax^2 - 3ay^2 - 4bx^2 + 4by^2 = 3a(x^2-y^2) - 4b(x^2-y^2)$
$= (x^2-y^2)(3a-4b)$
$= (x-y)(x+y)(3a-4b)$

Q18. $a^2 - b^2 = (a-b)(a+b)$

Q19. $x^2 - 4y^2 = (x-2y)(x+2y)$

Q20. $9x^2 - y^2 = (3x-y)(3x+y)$

Q21. $16x^2 - 25y^2 = (4x)^2 - (5y)^2 = (4x-5y)(4x+5y)$

Q22. $36x^2 - 25 = (6x-5)(6x+5)$

Q23. $1 - 36x^2 = (1-6x)(1+6x)$

Q24. $49a^2 - 4b^2 = (7a)^2 - (2b)^2 = (7a-2b)(7a+2b)$

Q25. $x^2y^2 - 1 = (xy-1)(xy+1)$

Q26. $4a^2b^2 - 16c^2 = (2ab)^2 - (4c)^2 = (2ab-4c)(2ab+4c)$

Q27. $3x^2 - 27y^2 = 3(x^2 - 9y^2)$
$= 3(x-3y)(x+3y)$

Q28. $45 - 5x^2 = 5(9 - x^2)$
$= 5(3-x)(3+x)$

Q29. $45a^2 - 20 = 5(9a^2 - 4)$
$= 5(3a-2)(3a+2)$

Q30. $(2x+y)^2 - 4 = (2x+y-2)(2x+y+2)$

Q31. $(3a-2b)^2 - 9 = (3a-2b-3)(3a-2b+3)$

Q**32**. $a^4 - b^4 = (a^2)^2 - (b^2)^2 = (a^2 - b^2)(a^2 + b^2)$

$$= (a-b)(a+b)(a^2+b^2)$$

Q**33**. $x^2 + 9x + 14 = (x+2)(x+7)$

Q**34**. $2x^2 + 7x + 3 = (2x+1)(x+3)$

Q**35**. $2x^2 + 11x + 14 = (2x+7)(x+2)$

Q**36**. $x^2 - 9x + 14 = (x-2)(x-7)$

Q**37**. $x^2 - 11x + 28 = (x-7)(x-4)$

Q**38**. $2x^2 - 7x + 3 = (2x-1)(x-3)$

Q**39**. $3x^2 - 17x + 20 = (3x-5)(x-4)$

Q**40**. $7x^2 - 18x + 8 = (7x-4)(x-2)$

Q**41**. $2x^2 - 7x - 15 = (2x+3)(x-5)$

Q**42**. $3x^2 + 11x - 20 = (3x-4)(x+5)$

Q**43**. $12x^2 - 11x - 5 = (4x-5)(3x+1)$

Q**44**. $6x^2 + x - 15 = (3x+5)(2x-3)$

Q**45**. $3x^2 + 13x - 10 = (3x-2)(x+5)$

Q**46**. $6x^2 - 11x + 3 = (3x-1)(2x-3)$

Q**47**. $36x^2 - 7x - 4 = (9x-4)(4x+1)$

Q**48**. $15x^2 - 14x - 8 = (5x+2)(3x-4)$

Q**49**. $6y^2 + 11y - 35 = (3y-5)(2y+7)$

Q**50**. $12x^2 + 17xy - 5y^2 = (4x-y)(3x+5y)$

Q**51**. (i) $x^2 + 3\sqrt{3}x + 6.$

$\Rightarrow\ a = 1,\ b = 3\sqrt{3},\ c = 6.$

$$\therefore\quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3\sqrt{3} \pm \sqrt{27 - 4(1)(6)}}{2(1)}$$

$$= \frac{-3\sqrt{3} \pm \sqrt{3}}{2}$$

$$\therefore\quad x = \frac{-3\sqrt{3} + \sqrt{3}}{2} \quad \text{or} \quad x = \frac{-3\sqrt{3} - \sqrt{3}}{2}$$

$\therefore\quad x = -\sqrt{3}$ or $-2\sqrt{3}$

$\therefore$ Factors are $(x + \sqrt{3})$ and $(x + 2\sqrt{3})$

(ii) $x^2 + 2\sqrt{5}x - 15.$

$\Rightarrow\ a = 1,\ b = 2\sqrt{5},\ c = -15.$

$\therefore\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-2\sqrt{5} \pm \sqrt{20 - 4(1)(-15)}}{2(1)}$

$= \dfrac{-2\sqrt{5} \pm \sqrt{80}}{2}$

$= \dfrac{-2\sqrt{5} \pm 4\sqrt{5}}{2}$

$\therefore\quad x = \dfrac{-2\sqrt{5} + 4\sqrt{5}}{2}$ or $x = \dfrac{-2\sqrt{5} - 4\sqrt{5}}{2}$

$x = \sqrt{5}$ or $-3\sqrt{5}$

$\therefore$ Factors are $(x - \sqrt{5})$ and $(x + 3\sqrt{5})$

(iii) $2x^2 - 5\sqrt{2}x - 6.$

$\Rightarrow\ a = 2,\ b = -5\sqrt{2},\ c = -6.$

$\therefore\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{5\sqrt{2} \pm \sqrt{50 - 4(2)(-6)}}{2(2)}$

$= \dfrac{5\sqrt{2} \pm \sqrt{98}}{4}$

$= \dfrac{5\sqrt{2} \pm 7\sqrt{2}}{4}$

$\therefore\quad x = \dfrac{5\sqrt{2} + 7\sqrt{2}}{4}$ or $x = \dfrac{5\sqrt{2} - 7\sqrt{2}}{4}$

$x = 3\sqrt{2}$ or $\dfrac{-\sqrt{2}}{2}$

$\therefore$ Factors are $(x - 3\sqrt{2})$ and $\left(x + \dfrac{\sqrt{2}}{2}\right)$

But since coefficient of x^2 is 2, one of the factors must contain $2x$. $\therefore\ x + \dfrac{\sqrt{2}}{2} = 0$

$\Rightarrow\ 2x + \sqrt{2} = 0$

$\therefore$ Factors are $(x - 3\sqrt{2})$ and $(2x + \sqrt{2})$

Q**52**. (i) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

(ii) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

(iii) $8x^3 + y^3 = ((2x)^3 + y^3) = (2x + y)(4x^2 - 2xy + y^2)$

Q**53**. (i) $27x^3 - y^3 = (3x)^3 - y^3 = (3x - y)(9x^2 + 3xy + y^2)$

(ii) $x^3 - 64 = x^3 - 4^3 = (x - 4)(x^2 + 4x + 16)$

(iii) $8x^3 - 27y^3 = (2x)^3 - (3y)^3 = (2x - 3y)(4x^2 + 6xy + 9y^2)$

Q**54**. (i) $8+27k^3=(2)^3+(3k)^3=(2+3k)(4-6k+9k^2)$

(ii) $64-125a^3=(4)^3-(5a)^3=(4-5a)(16+20a+25a^2)$

(iii) $27a^3+64b^3=(3a)^3+(4b)^3=(3a+4b)(9a^2-12ab+16b^2)$

Q**55**. (i) $a^3-8b^3c^3=a^3-(2bc)^3=(a-2bc)(a^2+2abc+4b^2c^2)$

(ii) $5x^3+40y^3=5(x^3+8y^3)$

$$=5(x^3+(2y)^3)$$

$$=5(x+2y)(x^2-2xy+4y^2)$$

(iii) $(x+y)^3-z^3=(x+y-z)[(x+y)^2+(x+y)z+z^2]$

Exercise 1.4

Q**1**. (i) $\dfrac{\cancel{8}^{4}\cancel{y}}{\cancel{2}y^{\cancel{3}2}}=\dfrac{4}{y^2}$ (ii) $\dfrac{\cancel{7}\,\cancel{a^6}^{a}\,\cancel{b^3}}{\cancel{14}_{2}\,\cancel{a^5}\,\cancel{b^4}_{b}}=\dfrac{a}{2b}$

(iii) $\dfrac{(2x)^2}{4x}=\dfrac{\cancel{4}\,\cancel{x^2}^{x}}{\cancel{4}\cancel{x}}=x$

(iv) $\dfrac{7y+2y^2}{7y}=\dfrac{\cancel{y}(7+2y)}{7\cancel{y}}=\dfrac{7+2y}{7}$

(v) $\dfrac{5ax}{15a+10a^2}=\dfrac{\cancel{5a}x}{\cancel{5a}(3+2a)}=\dfrac{x}{3+2a}$

Q**2**. (a) $\dfrac{2x}{5}+\dfrac{4x}{3}=\dfrac{6x}{15}+\dfrac{20x}{15}=\dfrac{26x}{15}$

(b) $\dfrac{3x}{5}-\dfrac{x}{2}=\dfrac{6x}{10}-\dfrac{5x}{10}=\dfrac{x}{10}$

(c) $\dfrac{2x+3}{4}+\dfrac{x}{3}=\dfrac{6x+9}{12}+\dfrac{4x}{12}=\dfrac{10x+9}{12}$

(d) $\dfrac{x+1}{4}+\dfrac{2x-1}{5}=\dfrac{5x+5}{20}+\dfrac{8x-4}{20}=\dfrac{13x+1}{20}$

(e) $\dfrac{3x-4}{6}-\dfrac{2x+1}{3}=\dfrac{3x-4}{6}-\dfrac{4x+2}{6}=\dfrac{-x-6}{6}$

(f) $\dfrac{3x-2}{6}-\dfrac{x-3}{4}=\dfrac{6x-4}{12}-\dfrac{3x-9}{12}=\dfrac{3x+5}{12}$

(g) $\dfrac{5x-1}{4}-\dfrac{2x-4}{5}=\dfrac{25x-5}{20}-\dfrac{8x-16}{20}=\dfrac{17x+11}{20}$

(h) $\dfrac{3x+5}{6}-\dfrac{2x+3}{4}-\dfrac{1}{12}=\dfrac{6x+10}{12}-\dfrac{6x+9}{12}-\dfrac{1}{12}$

$$=\frac{0}{12}=0$$

(i) $$\frac{3x-2}{4}+\frac{3}{5}-\frac{2x-1}{10}=\frac{15x-10}{20}+\frac{12}{20}-\frac{4x-2}{20}$$
$$=\frac{11x+4}{20}$$

(j) $$\frac{1}{3x}+\frac{1}{5x}=\frac{5}{15x}+\frac{3}{15x}=\frac{8}{15x}$$

(k) $$\frac{3}{4x}-\frac{5}{8x}=\frac{6}{8x}-\frac{5}{8x}=\frac{1}{8x}$$

(l) $$\frac{1}{x}+\frac{1}{x+3}=\frac{x+3+x}{x(x+3)}=\frac{2x+3}{x(x+3)}$$

(m) $$\frac{2}{x+2}+\frac{3}{x+4}=\frac{2(x+4)+3(x+2)}{(x+2)(x+4)}=\frac{5x+14}{(x+2)(x+4)}$$

(n) $$\frac{2}{x-2}+\frac{3}{2x-1}=\frac{2(2x-1)+3(x-2)}{(x-2)(2x-1)}=\frac{7x-8}{(x-2)(2x-1)}$$

(o) $$\frac{5}{3x-1}-\frac{2}{x+3}=\frac{5(x+3)-2(3x-1)}{(3x-1)(x+3)}=\frac{-x+17}{(3x-1)(x+3)}$$

(p) $$\frac{3}{2x-7}-\frac{1}{5x+2}=\frac{3(5x+2)-(2x-7)}{(2x-7)(5x+2)}=\frac{13x+13}{(2x-7)(5x+2)}$$

(q) $$\frac{2}{3x-5}-\frac{1}{4}=\frac{8-(3x-5)}{4(3x-5)}=\frac{13-3x}{4(3x-5)}$$

(r) $$\frac{5}{2x-1}-\frac{3}{x-2}=\frac{5(x-2)-3(2x-1)}{(2x-1)(x-2)}=\frac{-x-7}{(2x-1)(x-2)}$$

(s) $$\frac{x}{x-y}-\frac{y}{x+y}=\frac{x(x+y)-y(x-y)}{(x-y)(x+y)}=\frac{x^2+\cancel{xy}-\cancel{xy}+y^2}{(x-y)(x+y)}$$
$$=\frac{x^2+y^2}{x^2-y^2}$$

(t) $$\frac{3}{x}+\frac{4}{3y}-\frac{2}{3xy}=\frac{3(3y)+4(x)-2}{3xy}=\frac{9y+4x-2}{3xy}$$

(u) $$\frac{3}{x}-\frac{2}{x-1}-\frac{4}{x(x-1)}=\frac{3(x-1)-2(x)-4}{x(x-1)}=\frac{x-7}{x(x-1)}$$

Q3. (i) $$\frac{2z^2-4z}{2z^2-10z}=\frac{\cancel{2z}\,(z-2)}{\cancel{2z}\,(z-5)}=\frac{z-2}{z-5}$$

(ii) $$\frac{y^2+7y+10}{y^2-25}=\frac{\cancel{(y+5)}\,(y+2)}{\cancel{(y+5)}\,(y-5)}=\frac{y+2}{y-5}$$

(iii) $$\frac{t^2+3t-4}{t^2-3t+2}=\frac{(t+4)\,\cancel{(t-1)}}{(t-2)\,\cancel{(t-1)}}=\frac{t+4}{t-2}$$

(iv) $$\frac{x}{x^2-4}-\frac{1}{x+2}=\frac{x}{(x+2)(x-2)}-\frac{1}{x+2}$$
$$=\frac{x-(x-2)}{(x+2)(x-2)}=\frac{2}{(x+2)(x-2)}$$

(v) $$\frac{2}{a+3}-\frac{a+2}{a^2-9}=\frac{2}{a+3}-\frac{a+2}{(a+3)(a-3)}$$
$$=\frac{2(a-3)-(a+2)}{(a+3)(a-3)}=\frac{a-8}{(a+3)(a-3)}$$

(vi) $$\frac{x-1}{x^2-4}+\frac{1}{x-2}=\frac{x-1}{(x-2)(x+2)}+\frac{1}{x-2}$$
$$=\frac{x-1+x+2}{(x-2)(x+2)}=\frac{2x+1}{(x-2)(x+2)}$$

Q**4**. (i) $$\frac{10}{2x^2-3x-2}-\frac{2}{x-2}=\frac{10}{(2x+1)(x-2)}-\frac{2}{(x-2)}$$
$$=\frac{10-2(2x+1)}{(2x+1)(x-2)}$$
$$=\frac{8-4x}{(2x+1)(x-2)}=\frac{4(2-x)}{(2x+1)(x-2)}$$
$$=\frac{-4\,\cancel{(x-2)}}{(2x+1)\,\cancel{(x-2)}}$$

(ii) $$\frac{x+2}{2x^2-x-1}-\frac{1}{x-1}=\frac{x+2}{(2x+1)(x-1)}-\frac{1}{x-1}$$
$$=\frac{x+2-(2x+1)}{(2x+1)(x-1)}$$
$$=\frac{-x+1}{(2x+1)(x-1)}=\frac{-\cancel{(x-1)}}{(2x+1)\,\cancel{(x-1)}}=\frac{-1}{2x+1}$$

Q**5**. (i) $$\frac{1}{x^2-9}-\frac{2}{x^2-x-6}=\frac{1}{(x-3)(x+3)}-\frac{2}{(x-3)(x+2)}$$
$$=\frac{x+2-2(x+3)}{(x-3)(x+3)(x+2)}$$
$$=\frac{-x-4}{(x-3)(x+3)(x+2)}$$

(ii) $$\frac{3}{x^2+x-2}-\frac{2}{x^2+3x+2}=\frac{3}{(x+2)(x-1)}-\frac{2}{(x+2)(x+1)}$$
$$=\frac{3(x+1)-2(x-1)}{(x+2)(x-1)(x+1)}$$
$$=\frac{x+5}{(x+2)(x-1)(x+1)}$$

(iii) $$\frac{2}{6x^2-5x-4}-\frac{3}{9x^2-16}=\frac{2}{(3x-4)(2x+1)}-\frac{3}{(3x-4)(3x+4)}$$
$$=\frac{2(3x+4)-3(2x+1)}{(3x-4)(2x+1)(3x+4)}$$
$$=\frac{\cancel{6x}+8-\cancel{6x}-3}{(3x-4)(2x+1)(3x+4)}=\frac{5}{(3x-4)(2x+1)(3x+4)}$$

(iv) $$\frac{1}{xy-x^2}-\frac{1}{y^2-xy}=\frac{1}{x(y-x)}-\frac{1}{y(y-x)}$$
$$=\frac{y-x}{x(y-x)y}$$
$$=\frac{\cancel{(y-x)}}{\cancel{(y-x)}xy}=\frac{1}{xy}$$

Q6. (i) $$\frac{\frac{1}{2}+\frac{3}{4}}{\frac{1}{4}}=\frac{\frac{2}{4}+\frac{3}{4}}{\frac{1}{4}}=\frac{\frac{5}{4}}{\frac{1}{4}}=\frac{5}{4}\cdot\frac{4}{1}=5$$

(ii) $$\frac{\frac{2}{3}+\frac{5}{6}}{\frac{3}{8}}=\frac{\frac{4}{6}+\frac{5}{6}}{\frac{3}{8}}=\frac{\frac{9}{6}}{\frac{3}{8}}=\frac{\cancel{9}^{\cancel{3}^1}}{\cancel{6}_{\cancel{3}_1}}\cdot\frac{\cancel{8}^4}{\cancel{3}_1}=4$$

(iii) $$\frac{x-\frac{1}{x}}{1+\frac{1}{x}}=\frac{\frac{x^2-1}{x}}{\frac{x+1}{x}}=\frac{x^2-1}{x+1}=\frac{(x-1)\cancel{(x+1)}}{\cancel{(x+1)}}=x-1$$

Q7. (i) $$\frac{\frac{1}{x}+1}{\frac{1}{x}-1}=\frac{\frac{1+x}{x}}{\frac{1-x}{x}}=\frac{(1+x)}{\cancel{x}}\cdot\frac{\cancel{x}}{(1-x)}=\frac{1+x}{1-x}$$

(ii) $$\frac{\frac{1}{x^2}-4}{\frac{1}{x}-2}=\frac{\frac{1-4x^2}{x^2}}{\frac{1-2x}{x}}=\frac{(1-4x^2)}{x^{\cancel{2}}}\cdot\frac{\cancel{x}^1}{(1-2x)}$$
$$=\frac{\cancel{(1-2x)}(1+2x)}{x\cancel{(1-2x)}}=\frac{1+2x}{x}$$

(iii) $$\frac{x+y}{\frac{1}{x}+\frac{1}{y}}=\frac{x+y}{\frac{y+x}{xy}}=\frac{\cancel{(x+y)}}{1}\cdot\frac{xy}{\cancel{(x+y)}}=xy$$

Q8. (i) $$\frac{4y-\frac{3}{2}}{2}=\frac{\frac{8y-3}{2}}{2}=\frac{8y-3}{2}\times\frac{1}{2}=\frac{8y-3}{4}$$

(ii) $$\frac{2-\frac{1}{x}}{2}=\frac{\frac{2x-1}{x}}{2}=\frac{2x-1}{x}\cdot\frac{1}{2}=\frac{2x-1}{2x}$$

(iii) $$\frac{3x+\frac{1}{x}}{2}=\frac{\frac{3x^2+1}{x}}{2}=\frac{3x^2+1}{x}\cdot\frac{1}{2}=\frac{3x^2+1}{2x}$$

(iv) $$\frac{y+\frac{1}{4}}{\frac{1}{2}}=\frac{\frac{4y+1}{4}}{\frac{1}{2}}=\frac{4y+1}{\cancel{4}_2}\cdot\frac{\cancel{2}^1}{1}=\frac{4y+1}{2}$$

Q9. (i) $$\frac{z-\frac{1}{3}}{z-\frac{1}{2}}=\frac{\frac{3z-1}{3}}{\frac{2z-1}{2}}=\frac{3z-1}{3}\cdot\frac{2}{2z-1}=\frac{6z-2}{6z-3}$$

(ii) $$\frac{2x+\frac{1}{2}}{x+\frac{1}{4}}=\frac{\frac{4x+1}{2}}{\frac{4x+1}{4}}=\frac{\cancel{(4x+1)}}{\cancel{2}_1}\cdot\frac{\cancel{4}^2}{\cancel{(4x+1)}}=2$$

(iii) $$\frac{z-\frac{1}{2z}}{z-\frac{1}{3z}}=\frac{\frac{2z^2-1}{2z}}{\frac{3z^2-1}{3z}}=\frac{(2z^2-1)}{2\cancel{z}}\cdot\frac{3\cancel{z}}{(3z^2-1)}$$

$$=\frac{6z^2-3}{6z^2-2}$$

(iv) $$\frac{x-\frac{1}{x+1}}{x-1}=\frac{\frac{x(x+1)-1}{x+1}}{x-1}=\frac{x^2+x-1}{x+1}\cdot\frac{1}{x-1}$$

$$=\frac{x^2+x-1}{x^2-1}$$

Q10. (i) $$\frac{1+\frac{2}{x}}{\frac{x+2}{x-2}}=\frac{\frac{x+2}{x}}{\frac{x+2}{x-2}}=\frac{\cancel{(x+2)}}{x}\cdot\frac{x-2}{\cancel{(x+2)}}=\frac{x-2}{x}$$

(ii) $$\frac{2+\frac{1}{x}}{2x^2+x}=\frac{\frac{2x+1}{x}}{x(2x+1)}=\frac{\cancel{(2x+1)}}{x}\cdot\frac{1}{x\,\cancel{(2x+1)}}=\frac{1}{x^2}$$

(iii) $$\frac{x+\frac{2x}{x-2}}{1+\frac{4}{(x+2)(x-2)}}=\frac{\frac{x(x-2)+2x}{(x-2)}}{\frac{(x+2)(x-2)+4}{(x+2)(x-2)}}$$

$$=\frac{x^2-\cancel{2x}+\cancel{2x}}{(x-2)}\cdot\frac{(x+2)(x-2)}{x^2-\cancel{4}+\cancel{4}}$$

$$=\frac{\cancel{x^2}}{\cancel{(x-2)}}\cdot\frac{(x+2)\,\cancel{(x-2)}}{\cancel{x^2}}=x+2$$

Q11. (i) $$\frac{\left(\frac{a+b}{a-b}\right)-\left(\frac{a-b}{a+b}\right)}{1+\left(\frac{a-b}{a+b}\right)}=\frac{\frac{(a+b)(a+b)-(a-b)(a-b)}{(a-b)(a+b)}}{\frac{(a+\cancel{b})+(a-\cancel{b})}{(a+b)}}$$

$$=\frac{a^2+2ab+b^2-(a^2-2ab+b^2)}{(a-b)(a+b)}\cdot\frac{(a+b)}{2a}$$

$$=\frac{(\cancel{a^2}+2ab+\cancel{b^2}-\cancel{a^2}+2ab-\cancel{b^2})}{(a-b)(a+b)}\cdot\frac{(a+b)}{2a}$$

$$=\frac{{}^2\cancel{4}\,\cancel{a}b\,\cancel{(a+b)}}{(a-b)\,\cancel{(a+b)}\,\cancel{2}\,\cancel{a}}=\frac{2b}{a-b}$$

(ii) $$\frac{x+\frac{3}{x}}{x-\frac{9}{x^3}}=\frac{\frac{x^2+3}{x}}{\frac{x^4-9}{x^3}}=\frac{(x^2+3)}{\cancel{x}}\cdot\frac{x^{\cancel{3}\,2}}{(x^4-9)}$$

$$=\frac{\cancel{(x^2+3)}\,(x^2)}{(x^2-3)\,\cancel{(x^2+3)}}=\frac{x^2}{x^2-3}$$

(iii) $$\frac{9-\frac{1}{y^2}}{9+\frac{6}{y}+\frac{1}{y^2}} = \frac{\frac{9y^2-1}{y^2}}{\frac{9y^2+6y+1}{y^2}}$$

$$= \frac{9y^2-1}{\cancel{y^2}} \cdot \frac{\cancel{y^2}}{9y^2+6y+1}$$

$$= \frac{(3y-1)\cancel{(3y+1)}}{(3y+1)\cancel{(3y+1)}} = \frac{3y-1}{3y+1}$$

Q**12**. $$\frac{3x-5}{x-2} + \frac{1}{2-x} = \frac{(3x-5)(2-x)+(x-2)}{(x-2)(2-x)}$$

$$= \frac{6x-3x^2-10+5x+x-2}{(x-2)(2-x)}$$

$$= \frac{-3x^2+12x-12}{(x-2)(2-x)}$$

$$= \frac{-3(x^2-4x+4)}{(x-2)(2-x)}$$

$$= \frac{-3\cancel{(x-2)}\cancel{(x-2)}}{-\cancel{(x-2)}\cancel{(x-2)}} = 3.$$

Exercise 1.5

Q**1**. $ax^2+bx+c=(2x-3)(3x+4)$ for all x

$$=6x^2+8x-9x-12$$

$$=6x^2-x-12$$

$\therefore a=6, b=-1, c=-12$

Q**2**. $(3x-2)(x+5)=3x^2+px+q$ for all x

$$3x^2+15x-2x-10=3x^2+px+q$$

$$3x^2+13x-10=3x^2+px+q$$

$\therefore p=13, q=-10$

Q**3**. $x^2+6x+16=(x+a)^2+b$ for all x

$$x^2+6x+16=x^2+2ax+a^2+b$$

$\therefore \quad 2a=6 \Rightarrow a=3$

and $a^2+b=16$

$\therefore \quad 9+b=16 \Rightarrow b=7$

Q**4**. $x^2+4x-6=(x+a)^2+b$ for all x

$$x^2+4x-6=x^2+2ax+a^2+b$$

$\therefore \quad 2a=4 \Rightarrow a=2$

and $a^2+b=-6$

$\therefore \quad 4+b=-6 \Rightarrow b=-10$

Q**5**. $2x^2+5x+6=p(x+q)^2+r$ for all x

$$=p(x^2+2xq+q^2)+r$$

$$=px^2+2pqx+pq^2+r$$

$\therefore p=2$

and $2pq=5$

$$\therefore 2(2)q=5 \Rightarrow q=\frac{5}{4}$$

and $pq^2+r=6$

$$\therefore 2\left(\frac{5}{4}\right)^2+r=6$$

$$\frac{25}{8}+r=6 \Rightarrow r=6-\frac{25}{8}$$

$$=\frac{48-25}{8}=\frac{23}{8}$$

Q**6**. $(2x+a)^2 = 4x^2 + 12x + b$ for all values of x

$4x^2 + 4ax + a^2 = 4x^2 + 12x + b$

$\therefore 4a = 12 \Rightarrow a = 3$

and $a^2 = b \Rightarrow 9 = b$

Q**7**. $x^2 - 4x - 5 = (x-n)^2 - m$ for all values of x

$x^2 - 4x - 5 = x^2 - 2nx + n^2 - m$

$\therefore \quad -4 = -2n \Rightarrow n = 2$

and $-5 = n^2 - m$

$\therefore \quad -5 = 4 - m \Rightarrow m = 9$

Q**8**. (i) $V(x) = ax^3 + bx^2 + cx + d = (x+5)(x+3)(x+2)$ for all x

$$= (x^2 + 8x + 15)(x+2)$$

$$= x^3 + 2x^2 + 8x^2 + 16x + 15x + 30$$

$$= x^3 + 10x^2 + 31x + 30$$

$\therefore a = 1, b = 10, c = 31, d = 30.$

(ii) $S(x) = px^2 + qx + r = 2(x+3)(x+2) + 2(x+5)(x+3) + (x+5)(x+2)$

$$= 2(x^2 + 5x + 6) + 2(x^2 + 8x + 15) + (x^2 + 7x + 10)$$

$$= 5x^2 + 33x + 52$$

$\therefore p = 5, q = 33, r = 52$

Q**9**. $3(x-p)^2 + q = 3x^2 - 12x + 7$ for all x

$\therefore 3(x^2 - 2px + p^2) + q = 3x^2 - 12x + 7$

$\therefore 3x^2 - 6px + 3p^2 + q = 3x^2 - 12x + 7$

$\therefore \quad -6p = -12 \Rightarrow p = 2$

and $3p^2 + q = 7$

$\therefore \quad 3(2)^2 + q = 7 \Rightarrow q = -5$

Q**10**. $V(x) = x^3 + 12x^2 + bx + 30 = (x^2 + cx + 4)(x+a)$

$$= x^3 + ax^2 + cx^2 + acx + 4x + 4a$$

$$= x^3 + x^2(a+c) + x(ac+4) + 4a$$

$\therefore \quad a + c = 12$

and $b = ac + 4$

and $4a = 30 \Rightarrow a = \frac{30}{4} = \frac{15}{2}.$

$\therefore \quad a + c = 12 \Rightarrow c = 12 - a$

$$c = 12 - \frac{15}{2} = \frac{9}{2}$$

$\therefore \quad b = ac + 4 \Rightarrow b = \frac{15}{2} \cdot \frac{9}{2} + 4 = \frac{135 + 16}{4} = 37\tfrac{3}{4}$

Q11. $(x-4)^3 = x^3 + px^2 + qx - 64$ for all x

$(x-4)(x-4)(x-4) =$

$(x^2 - 8x + 16)(x-4) =$

$x^3 - 4x^2 - 8x^2 + 32x + 16x - 64 =$

$x^3 - 12x^2 + 48x - 64 =$

$\therefore\ p = -12,\ q = 48$

Q12. $(x+a)(x^2 + bx + 2) = x^3 - 2x^2 - x - 6$ for all x

$x^3 + bx^2 + 2x + ax^2 + abx + 2a =$

$x^3 + x^2(b+a) + x(2+ab) + 2a =$

$\therefore\quad b + a = -2$

and $2 + ab = -1$

and $2a = -6 \Rightarrow a = -3$

$\therefore\quad b + a = -2 \Rightarrow b - 3 = -2 \Rightarrow b = 1$

Q13. $(x-2)(x^2 + bx + c) = x^3 + 2x^2 - 5x - 6$ for all x

$x^3 + bx^2 + cx - 2x^2 - 2bx - 2c =$

$x^3 + x^2(b-2) + x(c-2b) - 2c =$

$\therefore\quad b - 2 = 2 \Rightarrow b = 4$

and $c - 2b = -5 \Rightarrow c - 2(4) = -5 \Rightarrow c = 3$

Q14. $(5a - b)x + b + 2c = 0$ for all x

$\therefore (5a-b)x + b + 2c = 0.x + 0$

$\therefore\quad 5a - b = 0 \Rightarrow b = 5a$

and $b + 2c = 0 \Rightarrow b = -2c$

$\therefore 5a = -2c$

$a = \dfrac{-2c}{5}$

Q15. $(4x + r)(x^2 + s) = 4x^3 + px^2 + qx + 2$ for all x

$4x^3 + 4xs + rx^2 + rs =$

$4x^3 + rx^2 + 4xs + rs =$

$\left.\begin{array}{l}\therefore\quad r = p \\ \text{and}\ \ 4s = q\end{array}\right\} \Rightarrow pq = r.4s = 4rs$

and $rs = 2$

$\therefore\quad pq = 4rs = 4(2) = 8$

Q16. $(x+s)(x-s)(ax+t)=ax^3+bx^2+cx+d$ for all x

$\therefore\ (x^2-s^2)(ax+t)=$

$\therefore\ ax^3+tx^2-as^2x-ts^2=$

$\therefore\ t=b$

$$\left.\begin{array}{l}\text{and } -as^2=c\\ \text{and } -ts^2=d\end{array}\right\}\therefore \frac{-a\cancel{s^2}}{-t\cancel{s^2}}=\frac{c}{d}$$

$\Rightarrow -ad=-ct$

but $t=b$ $\therefore -ad=-cb$

$\Rightarrow ad=cb$

Q17. $\dfrac{1}{(x+1)(x-1)}=\dfrac{A}{(x+1)}+\dfrac{B}{(x-1)}$ for all x

$\Rightarrow \dfrac{1}{(x+1)(x-1)}=\dfrac{A(x-1)+B(x+1)}{(x+1)(x-1)}$

$\Rightarrow\ 1=A(x-1)+B(x+1)$

$\Rightarrow\ 1=Ax-A+Bx+B$

$\Rightarrow\ 1=x(A+B)-A+B$

$\therefore\ 0.x+1=(A+B)x-A+B$

$\therefore\ A+B=0$

and $\underline{-A+B}=1$

adding: $2B=1$

$B=\dfrac{1}{2}$

since $A+B=0$ $\Rightarrow A=-B=-\dfrac{1}{2}$.

Q18. $\dfrac{1}{(x+2)(x-3)}=\dfrac{C}{(x+2)}+\dfrac{D}{(x-3)}$

$\dfrac{1}{(x+2)(x-3)}=\dfrac{C(x-3)+D(x+2)}{(x+2)(x-3)}$

$\Rightarrow\ 1=C(x-3)+D(x+2)$

$1=Cx-3C+Dx+2D$

$0.x+1=x(C+D)-3C+2D$

$\Rightarrow\ C+D=0$ (×3)

and $-3C+2D=1$

$\underline{3C+3D}=0$

$\therefore$ adding: $5D=1\Rightarrow$

$D=\dfrac{1}{5}$

since $C+D=0$ $\Rightarrow C=-D=-\dfrac{1}{5}$.

Q**19**. $$\frac{1}{(x+1)(x+4)}=\frac{A}{(x+1)}+\frac{B}{(x+4)}$$

$$\Rightarrow \frac{1}{(x+1)(x+4)}=\frac{A(x+4)+B(x+1)}{(x+1)(x+4)}$$

$\Rightarrow 1=Ax+4A+Bx+B$

$0.x+1=x(A+B)+4A+B$

$\therefore \quad A+B=0$

and $\quad \underline{4A+B=1}$

subtracting: $-3A=-1\Rightarrow A=\frac{1}{3}$

since $\quad A+B=0 \quad \Rightarrow B=-A=-\frac{1}{3}$.

Q**20**. $(x-3)^2$ is a factor of x^3+ax+b

$\Rightarrow (x-3)^2(x+k)=x^3+ax+b$

$\therefore\ (x^2-6x+9)(x+k)=x^3+ax+b$

$\therefore\ x^3+kx^2-6x^2-6kx+9x+9k=x^3+ax+b$

$\therefore\ x^3+x^2(k-6)+x(-6k+9)+9k=x^3+ax+b=x^3+\ 0.x^2+ax+b$

$\therefore\ k-6=0\Rightarrow k=6$

also $-6k+9=a \quad \therefore -6(6)+9=a\Rightarrow a=-27$

also $\quad 9k \quad =b \quad \therefore 9(6)=b\Rightarrow b=54.$

Q**21**. $(x-2)^2$ is a factor of x^3+px+q

$\Rightarrow\ (x-2)^2(x-k)=x^3+px+q$

$\therefore\ \ (x^2-4x+4)(x-k)=x^3+\ 0.x^2+px+q$

$\therefore\ \ x^3-kx^2-4x^2+4kx+4x-4k=x^3+\ 0.x^2+px+q$

$\therefore\ \ x^3+x^2(-k-4)+x(4k+4)-4k=$

$\therefore\ \ -k-4=0$

$\Rightarrow\ \ k=-4$

also $p=4k+4=4(-4)+4=-12$

$q=-4k=-4(-4)=+16.$

Q**22**. (x^2-4) is a factor of $x^3+cx^2+dx-12$

$\therefore (x^2-4)(x+k)=x^3+cx^2+dx-12$

$\therefore x^3+kx^2-4x-4k=x^3+cx^2+dx-12$

$\therefore \quad k=c$

also $\quad d=-4$

and $-4k=-12\Rightarrow k=3=c$

$\therefore (x^2-4)(x+3)=x^3+3x^2-4x-12$

$\Rightarrow (x-2)(x+2)(x+3)=x^3+3x^2-4x-12.$

Q23. (x^2+b) is a factor of $x^3-3x^2+bx-15$

$\Rightarrow (x^2+b)(x+k)=x^3-3x^2+bx-15$

$\therefore x^3+kx^2+bx+bk=x^3-3x^2+bx-15$

$\Rightarrow \quad k=-3$

also $bk=-15 \Rightarrow b=\dfrac{-15}{k}=\dfrac{-15}{-3}=5.$

Q24. x^2-px+9 is a factor of x^3+ax+b

$\therefore (x^2-px+9)(x+k)=x^3+0.x^2+ax+b$

$\therefore x^3+kx^2-px^2-pkx+9x+9k=x^3+0.x^2+ax+b$

$\therefore x^3+x^2(k-p)+x(-pk+9)+9k=x^3+0.x^2+ax+b$

$\therefore \quad k-p=0 \Rightarrow k=p$

$-pk+9=a \Rightarrow -p(p)+9=a$

$\therefore a=9-p^2$

also $\quad b=9k \Rightarrow b=9p$

$a+b=17$

$\Rightarrow \quad 9-p^2+9p=17$

$-p^2+9p-8=0$

$p^2-9p+8=0$

$(p-8)(p-1)=0$

$\therefore p=8, 1$

Q25. x^2-kx+1 is a factor of ax^3+bx+c

$$
\begin{array}{r|l}
 & ax+ak \\
\hline
x^2-kx+1 & \cancel{ax^3}+\quad +bx+c \\
 & \underline{\cancel{ax^3}-akx^2+ax} \\
 & \quad akx^2+bx-ax+c \\
 & \quad \cancel{akx^2}+(b-a)x+c \\
 & \quad \underline{\cancel{akx^2}-ak^2x\quad +ak} \\
 & \quad\quad (b-a)x+ak^2x+c-ak \\
 & \quad\quad (b-a+ak^2)x+c-ak \quad \text{(remainder)}
\end{array}
$$

Since x^2-kx+1 is a factor, there can be no remainder.

$\therefore \quad (b-a+ak^2)x^2+c-ak=0 \quad$ for all x

$\Rightarrow \quad b-a+ak^2=0$

and $c-ak=0 \Rightarrow k=\dfrac{c}{a}$

$\therefore \quad b-a+a\left(\dfrac{c}{a}\right)^2=0$

$\therefore \quad b-a+\dfrac{c^2}{a}=0 \Rightarrow ab-a^2+c^2=0$

$\Rightarrow \quad c^2=a^2-ab=a(a-b)$

Q26. $(x-a)^2$ is a factor of $x^3+3px+c$

i.e. $x^2-2ax+a^2$ is a factor of $x^3+3px+c$.

$$\begin{array}{r|l} & x+2a \\ \hline \therefore x^2-2ax+a^2 & \cancel{x^3} \qquad\quad +3px+c \\ & \underline{\cancel{x^3}-2ax^2+a^2x} \\ & \qquad 2ax^2+3px-a^2x+c \\ & \qquad \cancel{2ax^2}+(3p-a^2)x+c \\ & \qquad \underline{\cancel{2ax^2}-4a^2x+2a^3} \\ & \qquad\qquad (3p-a^2)x+4a^2x+c-2a^3 \text{ (remainder)} \end{array}$$

Since $x^2-2ax+a^2$ is a factor, there can be no remainder.

$\therefore (3p-a^2+4a^2)x+c-2a^3=0$ for all x

$\therefore 3p-a^2+4a^2=0 \Rightarrow 3p=-3a^2 \Rightarrow p=-a^2$

also, $c-2a^3=0 \Rightarrow c=2a^3$.

Q27. x^2+ax+b is a factor of x^3-k.

$$\begin{array}{r|l} & x-a \\ \hline \therefore\ x^2+ax+b & \cancel{x^3} \qquad\quad -k \\ & \underline{\cancel{x^3}+ax^2+bx} \\ & \quad -\cancel{ax^2}-bx-k \\ & \quad \underline{-\cancel{ax^2}-a^2x-b} \\ & \qquad -bx+a^2x-k+b \\ & \qquad x(-b+a^2)-k+ba \quad \text{(remainder)} \end{array}$$

Since x^2+ax+b is a factor, there can be no remainder.

$\therefore (-b+a^2)=0 \Rightarrow b=a^2$

(i) also, $-k+ba=0 \Rightarrow k=ab=a.a^2=a^3$

(ii) Since $b=a^2 \Rightarrow b^3=(a^2)^3=a^6=k^2$.

Q28.

$$\begin{array}{r|l} & 2x-1 \\ \hline 2x-\sqrt{3} & \cancel{4x^2}-2(1+\sqrt{3})x+\sqrt{3} \\ & \underline{\cancel{4x^2}-2\sqrt{3}x} \\ & \quad -2(1+\sqrt{3})x+2\sqrt{3}x+\sqrt{3} \\ & \quad -2x-\cancel{2\sqrt{3}x}+\cancel{2\sqrt{3}x}+\sqrt{3} \\ & \quad -\cancel{2x}+\cancel{\sqrt{3}} \\ & \quad \underline{-\cancel{2x}+\cancel{\sqrt{3}}} \\ & \qquad\quad 0 \end{array}$$

$\therefore 2x-\sqrt{3}$ is a factor

and the second factor is $2x-1$.

Q29.
$$5x+3 = Ax(x+3)+Bx(x-1)+C(x-1)(x+3) \text{ for all } x$$
$$5x+3 = Ax^2+3Ax+Bx^2-Bx+C(x^2+3x-x-3)$$
$$= Ax^2+3Ax+Bx^2-Bx+Cx^2+3Cx-Cx-3C$$
$$= Ax^2+3Ax+Bx^2-Bx+Cx^2+2Cx-3C$$
$$= (A+B+C)x^2+(3A-B+2C)x-3C$$

$\therefore\ A+B+C=0$

also $3A-B+2C=5$

and $-3C=3 \Rightarrow C=-1$

$\therefore\quad A+B=1$

$\underline{3A-B=7}$

adding: $4A \quad = 8 \Rightarrow A=2$

since $A+B=1 \Rightarrow B=1-A=1-2=-1.$

Exercise 1.6

Q1. (i)
$$3x-2y=4$$
$$3x = 4+2y$$
$$x = \frac{4+2y}{3}$$

(ii)
$$2x-b=4c$$
$$2x = 4c+b$$
$$x = \frac{4c+b}{2}$$

(iii)
$$5x-4 = \frac{y}{2}$$
$$5x = \frac{y}{2}+4$$
$$x = \frac{\frac{y}{2}+4}{5} = \frac{y+8}{10}$$

(iv)
$$5(x-3)=2y$$
$$x-3 = \frac{2y}{5}$$
$$x = \frac{2y}{5}+3 = \frac{2y+15}{5}$$

(v)
$$3y = \frac{x}{3}-2$$
$$\frac{-x}{3} = -3y-2$$
$$\frac{x}{3} = 3y+2$$
$$x = 9y+6$$

(vi) $xy = xz + yz$

$xy - xz = yz$

$x(y - z) = yz$

$x = \dfrac{yz}{y - z}$

Q**2**. (i) $2x - \dfrac{y}{3} = \dfrac{1}{3}$

$2x = \dfrac{1}{3} + \dfrac{y}{3} = \dfrac{y+1}{3}$

$x = \dfrac{y+1}{6}$

(ii) $z = \dfrac{y - 2x}{3}$

$3z = y - 2x$

$2x = y - 3z$

$x = \dfrac{y - 3z}{2}$

(iii) $\dfrac{a}{x} - b = c$

$a - bx = cx$

$-cx - bx = -a$

$cx + bx = a$

$x(c + b) = a$

$x = \dfrac{a}{b + c}$

Q**3**. (a) $V = \pi r^2 h$

$\Rightarrow \pi r^2 h = V$

$\Rightarrow r^2 = \dfrac{V}{\pi h}$

$r = \sqrt{\dfrac{V}{\pi h}}$

(b) $A = 2\pi rh$

$\Rightarrow 2\pi rh = A$

$r = \dfrac{A}{2\pi h}$

(c) $r = \sqrt{\dfrac{V}{\pi h}}$ and $r = \dfrac{A}{2\pi h}$

$$\Rightarrow \sqrt{\frac{V}{\pi h}} = \frac{A}{2\pi h}$$

$$\therefore \frac{V}{\pi h} = \frac{A^2}{4\pi^2 h^2}$$

$$\Rightarrow A^2 = \frac{4\pi^2 h^2 V}{\pi h} = 4\pi h V$$

Q**4**. (a) $A_{\text{circle}} = \pi r^2$

(b) The side of the square $= 2r$

$\Rightarrow A_{\text{square}} = l^2 = (2r)^2 = 4r^2$

(c) $A_{\text{corners}} = 4r^2 - \pi r^2 = (4-\pi)r^2$

(d) area of new square $= (4r)^2 = 16r^2$

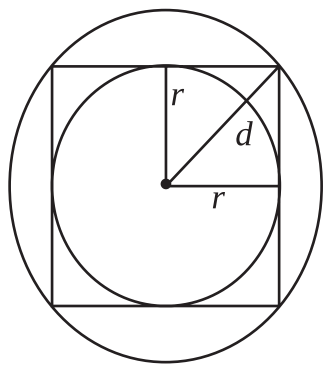
area of new circle $= \pi\left(\dfrac{r}{2}\right)^2 = \dfrac{\pi r^2}{4}$

$$\Rightarrow \text{area of new shaded section} = 16r^2 - \frac{\pi r^2}{4}$$

$$= r^2\left(16 - \frac{\pi}{4}\right)$$

$$= \frac{r^2}{4}(64 - \pi)$$

(e) To find the radius of the outer circle we need to find the distance from the centre of the circle to a corner (vertex) of the square.



$$d = \sqrt{r^2 + r^2} = \sqrt{2r^2} = \sqrt{2}r.$$

$\Rightarrow$ Area of outer circle $= \pi d^2$

$$= \pi\left(\sqrt{2}r\right)^2$$

$$= 2\pi r^2$$

= twice the area of the inner circle.

Q**5**. (i)

$$f^1 = \frac{fc}{c-u}$$

$$\Rightarrow f^1(c-u) = fc$$

$$c-u = \frac{fc}{f^1}$$

$$-u = \frac{fc}{f^1} - c$$

$$u = c - \frac{fc}{f^1} = \frac{f^1c - fc}{f^1} = \frac{c\left(f^1 - f\right)}{f^1}$$

(ii)

$$f^1 = \frac{fc}{c-u}$$

$$f^1(c-u) = fc$$

$$f^1c - f^1u = fc$$

$$f^1c - fc = f^1u$$

$$c(f^1 - f) = f^1u$$

$$c = \frac{f^1u}{f^1 - f}$$

Q**6**. (i)

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$T^2 = 4\pi^2\frac{l}{g}$$

$$l = \frac{gT^2}{4\pi^2}$$

(ii)

$$T = 3, \quad g = 10. \qquad \Rightarrow \qquad l = \frac{10\,.\,3^2}{4\pi^2} = \frac{90}{39.48} \simeq 2.3\,\text{m}$$

Q**7**. (i)

$$\frac{x}{y} = \frac{a+b}{a-b}$$

$$x(a-b) = y(a+b)$$

$$ax - bx = ay + by$$

$$ax - ay = bx + by$$

$$a(x-y) = b(x+y)$$

$$a = \frac{b(x+y)}{(x-y)}$$

(ii) $bc - ac = ac$

$-ac - ac = -bc$

$-2ac = -bc$

$$a = \frac{-bc}{-2c} = \frac{b}{2}$$

Q**8**. (i) $y = \frac{3(u-v)}{4}$

$4y = 3u - 3v$

$3v = 3u - 4y$

$$v = \frac{3u - 4y}{3}$$

(ii) $s = \frac{t}{2}(u+v)$

$2s = tu + tv$

$-tv = tu - 2s$

$tv = 2s - tu$

$$v = \frac{2s - tu}{t}$$

Q**9**.

$$A = P\left(1 + \frac{i}{100}\right)^3$$

$$P\left(1 + \frac{i}{100}\right)^3 = A$$

$$\left(1 + \frac{i}{100}\right)^3 = \frac{A}{P}$$

$$1 + \frac{i}{100} = \sqrt[3]{\frac{A}{P}}$$

$$100 + i = 100\sqrt[3]{\frac{A}{P}}$$

$$i = 100\sqrt[3]{\frac{A}{P}} - 100$$

$P = 2500, \qquad A = 2650$

$$\therefore \quad i = 100\sqrt[3]{\frac{2650}{2500}} - 100$$

$$= 100(1.0196) - 100$$

$$= 1.961$$

$$\therefore \quad i = 2\%$$

Q10. (i)

$$d = \sqrt{\frac{a-b}{ac}}$$

$$d^2 = \frac{a-b}{ac}$$

$$acd^2 = a-b$$

$$c = \frac{a-b}{ad^2}$$

(ii)

$$b = \frac{2c-1}{c-1}$$

$$\Rightarrow b(c-1) = 2c-1$$

$$bc - b = 2c - 1$$

$$bc - 2c = b - 1$$

$$c(b-2) = b-1$$

$$c = \frac{b-1}{b-2}$$

Q11. (i) From Pythagoras: $h^2 + r^2 = 15^2$

$$h^2 = 15^2 - r^2$$

$$h = \sqrt{15^2 - r^2}$$

(ii) at $r = 5$cm: $h = \sqrt{15^2 - 5^2}$

$$= \sqrt{225-25} = \sqrt{200}$$

$$= 10\sqrt{2}\text{ cm}$$

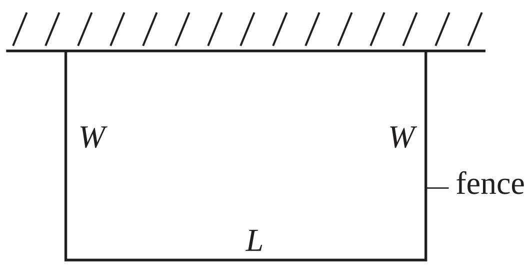
(iii) $r = \frac{15}{2}$: $h = \sqrt{15^2 - \left(\frac{15}{2}\right)^2}$

$$= \sqrt{168.75} = 12.99\text{cm}$$

$$h = 13\text{cm}$$

Q12. (i) $2W + L = 300$

$$L = 300 - 2W$$



(ii) $A = L \times W$

$$= (300-2W).W = 300W - 2W^2$$

(iii) $10{,}000 = 300W - 2W^2$

$2W^2 - 300W + 10{,}000 = 0$

$W^2 - 150W + 5{,}000 = 0$

$(W - 50)(W - 100) = 0$

$\Rightarrow \quad W = 50 \quad \text{or} \quad W = 100$

hence, $L = 300 - 2(50)$ or $L = 300 - 2(100)$

$= 200 \qquad\qquad = 100$

answer: $(50, 200)$ or $(100, 100)$

Exercise 1.7

Q**1**. (a) $4, 7, 10, 13, 16 \ldots$ constant 1^{st} difference $\Rightarrow$ linear

(b) $-2, 2, 6, 10, 14 \ldots$ constant 1^{st} difference $\Rightarrow$ linear

(c) $-4, -3, 0, 5, 12$

$+1, +3, +5, +7$

$+2, +2, +2 \ldots.$ constant 2^{nd} difference $\Rightarrow$ quadratic

(d) $2, 1, -2, -7, -14, -23 \ldots$

$-1, -3, -5, -7,$

$-2, -2, -2 \ldots$ constant 2^{nd} difference $\Rightarrow$ quadratic

(e) $2, 7, 22, 47$

$+5, +15, +25$

$+10, +10 \ldots.$ constant 2^{nd} difference $\Rightarrow$ quadratic

(f) $3, 1, -5, -15, -29, \ldots.$

$-2, -6, -10, -14$

$-4, -4, -4, \ldots$ constant 2^{nd} difference $\Rightarrow$ quadratic

(g) $1, -4, -19, -44, -79 \ldots.$

$-5, -15, -25, -35$

$-10, -10, -10 \ldots$ constant 2^{nd} difference $\Rightarrow$ quadratic

(h) $3, -2, -7, -12, -17 \ldots$

$-5, -5, -5, -5 \ldots$ constant 1^{st} difference $\Rightarrow$ linear

(i) $0, 3, 12, 27, 48$

$+3, +9, +15, +21$

$+6, +6, +6 \ldots.$ constant 2^{nd} difference $\Rightarrow$ quadratic

(j) $5, 17, 37, 65, 101$

$+12, +20, +28, +36$

$+8, +8, +8 \ldots$ constant 2^{nd} difference $\Rightarrow$ quadratic.

Q**2**. (a) $-1, 3, 15, 35, 63$

first differences $= +4, +12, +20, +28$

second differences $= +8, +8, +8.$

$\Rightarrow$ quadratic pattern of the form $ax^2 + bx + c$

also, $2a = 8 \Rightarrow a = 4.$

$\therefore 4x^2 + bx + c$

let $x = 1 \Rightarrow \quad 4(1)^2 + b(1) + c = -1$

$\Rightarrow \quad b + c = -1 - 4 = -5$

let $x = 2 \Rightarrow 4(2)^2 + b(2) + c = 3$

$\Rightarrow \quad 2b + c = 3 - 16 = -13$

using simultaneous equations: $2b + c = -13$

$\underline{b + c = -5}$

$b \quad = -8$

$\Rightarrow -8 + c = -5$

$\Rightarrow \quad c = -5 + 8 = 3$

$\therefore ax^2 + bx + c = 4x^2 - 8x + 3$ for $x = 1, 2, 3, ...$

Note also we could let $x = 0 \Rightarrow 4(0)^2 + b(0) + c = -1$

$\Rightarrow \quad c = -1$

let $x = 1 \Rightarrow \quad 4(1)^2 + b(1) + c = 3$

$4 + b + c = 3$

$4 + b - 1 = 3$

$b \quad = 0.$

$\therefore ax^2 + bx + c = 4x^2 - 1$ for $x = 0, 1, 2, ...$

(b) $4, 3, 0, -5, -12, -21, -32$

first difference $= -1, -3, -5, -7, -9, -11$

second differnce $= -2, -2, -2, -2$

$\Rightarrow$ quadratic pattern of the form $ax^2 + bx + c$

also, $2a = -2 \Rightarrow a = -1.$

$\therefore -x^2 + bx + c$

let $x = 0 \quad \Rightarrow \quad -(0)^2 + b(0) + c = 4$

$c = 4$

let $x = 1 \quad \Rightarrow \quad -(1)^2 + b(1) + c = 3$

$-1 + b + 4 = 3$

$b \quad = 0$

$\therefore ax^2 + bx + c = -x^2 + 4$ is the pattern for $x = 0, 1, 2, ...$

Q**3**. (i) $2, 7, 12, 17, 22, \ldots\ldots$

first difference $= 5$, a constant $\Rightarrow$ a linear pattern.

$\therefore\ f(x) = ax + b = 5x + b.$

Let $x = 0$ be the 1^{st} term of the pattern.

$\Rightarrow\ f(0) = 5(0) + b = 2$

$\therefore\ b = 2$

$\therefore\ f(x) = 5x + 2$ for $x = 0, 1, 2, \ldots$

(ii) $-6, -2, 2, 6, 10 \ldots$

first difference $= 4$, a constant $\Rightarrow$ a linear pattern.

$\therefore\ f(x) = ax + b = 4x + b$

Let $x = 0$ be the 1^{st} term of the pattern.

$\Rightarrow\ f(0) = 4(0) + b = -6$

$\therefore\ b = -6.$

$\therefore\ f(x) = 4x - 6$ for $x = 0, 1, 2, \ldots$

(iii) $3, 2, 1, 0, -1, -2, \ldots$

first difference $= -1$, a constant $\Rightarrow$ a linear relationship

$\therefore\ f(x) = ax + b = -x + b$

Let $x = 0$ be the 1^{st} term of the pattern.

$\Rightarrow\ f(0) = -(0) + b = 3$

$\therefore\ b = 3$

$\therefore\ f(x) = -x + 3$

$= 3 - x.$

(v) $3, 3.5, 4, 4.5, 5, \ldots$

first difference $= 0.5$, a constant $\Rightarrow$ a linear relationship.

$\therefore\ f(x) = ax + b = 0.5x + b$

Let $x = 0$ be the first term of the pattern.

$\therefore\ f(0) = 0.5(0) + b = 3$

$\Rightarrow\ b = 3$

$\therefore\ f(x) = 0.5(x) + 3$

$= \dfrac{x}{2} + 3$ for $x = 0, 1, 2, \ldots$

(vi) $-1, -0.8, -0.6, -0.4, -0.2, \ldots$

first difference $= 0.2$, a constant $\Rightarrow$ a linear relationship.

$\therefore\ f(x) = ax + b = 0.2x + b$

Let $x = 0$ be the first term of the pattern.

$\therefore\ f(0) = 0.2(0) + b = -1$

$\therefore\ b = -1$

$\therefore\ f(x) = 0.2x - 1$ for $x = 0, 1, 2, \ldots$

Q**4**. $11, 13, 15, 17, 19, \ldots$

first difference $= 2$, a constant $\Rightarrow$ a linear relationship.

$\therefore\ f(x) = ax + b = 2x + b$

Let $x = 3$ be the first term of the pattern.

$\Rightarrow\ f(3) = 2(3) + b = 11$

$\Rightarrow\ b = 5$

$\therefore\ f(x) = 2x + 5$ for $x = 3, 4, 5, \ldots$

Q**5**. $1, 3, 5, 7, 9, \ldots$

first difference $= 2$, a constant $\Rightarrow$ a linear relationship.

$\therefore\ f(x) = ax + b = 2x + b$

Let $x = -2$ be the first term of the pattern.

$\therefore\ f(-2) = 2(-2) + b = 1$

$b = 1 + 4 = 5.$

$\therefore\ f(x) = 2x + 5$ for $x = -2, -1, 0, \ldots$

Q**6**. (a) $3, 6, 9, \ldots$

a first difference $= 3$ (a constant) $\Rightarrow$ a linear pattern.

$\Rightarrow f(x) = ax + b = 3x + b$

Let $x = 1$ be the first element of the pattern.

$\therefore\ f(1) = 3(1) + b = 3$

$\Rightarrow\ b = 0.$

$\therefore\ f(x) = 3x$ for $x = 1, 2, 3, \ldots$

$\Rightarrow$ for the 15^{th} element, $x = 15$.

$\therefore f(15) = 3(15) = 45$ matchsticks are needed.

(b) $4, 8, 12, \ldots$

a first difference $= 4$ (a constant) $\Rightarrow$ a linear pattern.

$\Rightarrow f(x) = ax + b = 4x + b$

Let $x = 1$ be the first element of the pattern.

$\therefore\ f(1) = 4(1) + b = 4$

$\Rightarrow\ b = 0.$

$\therefore\ f(x) = 4x$ for $x = 1, 2, 3, \ldots$

$\Rightarrow$ for the 15^{th} element, $x = 15$.

$\therefore f(15) = 4(15) = 60$ matchsticks are needed.

(c) $3, 5, 7, \ldots$

a first difference $= 2$ (constant) $\Rightarrow$ a linear pattern.

$\Rightarrow f(x) = ax + b = 2x + b$

Let $x = 1$ be the first element of the pattern.

$\therefore\ f(1) = 2(1) + b = 3$

$\Rightarrow\ b = 1$

$\therefore\ f(x) = 2x + 1$ for $x = 1, 2, 3, \ldots$

$\Rightarrow$ For the 15^{th} element, $x = 15$.

$\therefore\ f(15) = 2(15) + 1 = 31$ matchsticks are needed.

Q7. Plan $A = 35x + 70$

Plan $B = 24x + 125$

Both plans repay the same amount if

$$35x + 70 = 24x + 125$$

$$\Rightarrow \quad 11x = 55$$

$$x = 5 \text{ months.}$$

Q8. 4, 7, 14, 25, 40

first difference: 3, 7, 11, 15

second difference: 4, 4, 4 $\Rightarrow$ a quadratic pattern, $f(t) = at^2 + bt + c$

$$\therefore 2a = 4 \Rightarrow a = 2.$$

$$\therefore f(t) = 2t^2 + bt + c$$

Let $t = 1$ $\quad \therefore f(1) = 2(1)^2 + b(1) + c = 4$ (i.e. after 1 hour, there were 4 bacteria)

$$\Rightarrow \quad b + c = 2.$$

Let $t = 2$ $\quad \therefore f(2) = 2(2)^2 + b(2) + c = 7$

$$\Rightarrow \quad 2b + c = -1$$

$$\underline{b + c = 2}$$

Solving simultaneous equations: $\quad b \quad = -3$

since $b + c = 2$

$\Rightarrow \quad -3 + c = 2$

$c = 5.$

$\therefore f(t) = 2t^2 - 3t + 5 \quad$ for $\quad t = 1, 2, 3, \ldots$

Note : If we consider that the number of bacteria at the start (i.e. $t = 0$) was 4, then $f(t) = 2t^2 + bt + c$ gives:

at $t = 0$, $\quad f(0) = 2(0)^2 + b(0) + c = 4 \quad$ (i.e. at the start there were 4 bacteria)

$$\Rightarrow \quad c = 4$$

at $t = 1$, $\quad f(1) = 2(1)^2 + b(1) + c = 7$

$$\Rightarrow 2 + b + 4 = 7$$

$$\Rightarrow \quad b = 1$$

$\therefore f(t) = 2t^2 + t + 4 \quad$ for $\quad t = 0, 1, 2, 3, \ldots.$

when is $f(t) = 529$, assuming $f(t) = 2t^2 + t + 4$?

if $t = 10 \Rightarrow 2(10)^2 + 10 + 4 = 214$ too small

$t = 15 \Rightarrow 2(15)^2 + 15 + 4 = 469$ too small

$t = 16 \Rightarrow 2(16)^2 + 16 + 4 = 532$ too small

$t = 17 \Rightarrow 2(17)^2 + 17 + 4 = 599$ too large.

$\therefore$ In the 16^{th} hour, the number of bacteria was 529.

Exercise 1.8

Q**1**. (i) $y = 2x^2 + 2x - 1$ is not linear because the highest power is not 1.

(ii) $y = 2(x-1)^{-1}$ is not linear because the highest power of x is not 1.

(iii) $y^2 = 3x + 4$

$\Rightarrow \; y = \sqrt{3x+4}$ is not linear because the highest power of x is not 1.

$= (3x+4)^{\frac{1}{2}}$

Q**2**. (i) Solve $5x - 3 = 32$

$\Rightarrow 5x = 35$

$x = \frac{35}{5} = 7$

(ii) Solve $3x + 2 = x + 8$

$\Rightarrow \quad 3x - x = 8 - 2$

$2x = 6$

$x = 3$

(iii) Solve $2 - 5x = 8 - 3x$

$\Rightarrow \quad 3x - 5x = 8 - 2$

$-2x = 6$

$x = \frac{6}{-2} = -3$

Q**3**. (i) Solve $2(x-3) + 5(x-1) = 3$

$\Rightarrow 2x - 6 + 5x - 5 = 3$

$\Rightarrow \quad 7x = 3 + 11 = 14$

$x = 2$

(ii) Solve $2(4x-1) - 3(x-2) = 14$

$\Rightarrow \quad 8x - 2 - 3x + 6 = 14$

$5x = 14 - 4 = 10$

$x = 2$

(iii) Solve $3(x-1) - 4(x-2) = 6(2x+3)$

$\Rightarrow 3x - 3 - 4x + 8 = 12x + 18$

$-x - 12x = 18 - 5$

$-13x = 13$

$x = -1$

(iv) Solve $3(x+5) + 2(x+1) - 3x = 22$

$\cancel{3x} + 15 + 2x + 2 - \cancel{3x} = 22$

$2x = 22 - 17$

$2x = 5$

$x = \frac{5}{2} = 2.5$

Q4. (i) $\dfrac{2x+1}{5}=1$

$\Rightarrow 2x+1=5$

$2x=4$

$x=2$

(ii) $\dfrac{3x-1}{4}=8$

$\Rightarrow 3x-1=32$

$3x=33$

$x=11$

(iii) $\dfrac{x-3}{4}=\dfrac{x-2}{5}$

$\Rightarrow 5(x-3)=4(x-2)$

$5x-15=4x-8$

$5x-4x=15-8$

$x=7$

Q5. (i) $\dfrac{2a}{3}-\dfrac{a}{4}=\dfrac{5}{6}$

$\Rightarrow$ multiplying each term by 12: $4(2a)-3(a)=2(5)$

$\Rightarrow 8a-3a=10$

$5a=10$

$a=2$

(ii) $\dfrac{b+2}{4}-\dfrac{b-3}{3}=\dfrac{1}{2}$

$\Rightarrow$ multiplying each term by 12: $3(b+2)-4(b-3)=6$

$\Rightarrow 3b+6-4b+12=6$

$-b=6-18$

$-b=-12$

$b=12$

(iii) $\dfrac{3c-1}{6}-\dfrac{c-3}{4}=\dfrac{4}{3}$

$\Rightarrow$ multiplying each term by 12: $2(3c-1)-3(c-3)=4(4)$

$\Rightarrow 6c-2-3c+9=16$

$3c=16-7$

$3c=9$

$c=3$

Q**6**. (i) $\frac{x-2}{5}+\frac{2x-3}{10}=\frac{1}{2}$

multiplying each term by 10 we get : $2(x-2)+(2x-3)=5(1)$

$$\Rightarrow 2x-4+2x-3=5$$
$$4x=5+7$$
$$4x=12$$
$$x=3$$

(ii) $\frac{3y-12}{5}+3=\frac{3(y-5)}{2}$

multiplying each term by 10 we get : $2(3y-12)+10(3)=5.3(y-5)$

$$\Rightarrow 6y-24+30=15y-75$$
$$\therefore \quad 6y-15y=24-30-75$$
$$-9y=-81$$
$$y=\frac{-81}{-9}=+9$$

(iii) $\frac{3p-2}{6}-\frac{3p+1}{4}=\frac{2}{3}$

multiplying each term by 12 we get : $2(3p-2)-3(3p+1)=4(2)$

$$\Rightarrow 6p-4-9p-3=8$$
$$-3p=8+7$$
$$-3p=15$$
$$p=-5$$

(iv) $\frac{3r-2}{5}-\frac{2r-3}{4}=\frac{1}{2}$

multiplying each term by 20 we get : $4(3r-2)-5(2r-3)=10(1)$

$$\Rightarrow 12r-8-10r+15=10$$
$$2r=10-7$$
$$2r=3$$
$$r=\frac{3}{2}=1.5$$

Q**7**. (i) $\frac{3}{4}(2x-1)-\frac{2}{3}(4-x)=2.$

multiplying each term by 12 we get : $3.3(2x-1)-4.2(4-x)=12.2$

$$\Rightarrow 18x-9-32+8x=24$$
$$26x=24+41$$
$$26x=65$$
$$x=\frac{65}{26}=2.5$$

(ii) $\frac{2}{3}(x-1)-\frac{1}{5}(x-3)=x+1$

multiplying each term by 15 we get : $5.2(x-1)-3.1(x-3)=15x+15$

$$\Rightarrow 10x-10-3x+9=15x+15$$

$$7x-15x=15+1$$

$$-8x=16$$

$$x=-2.$$

Exercise 1.9

Q**1**. (i) $3x-2y=8$, $x+y=6$ $\Rightarrow$ $3x-2y=8$, $2x+2y=12$

(adding) $5x=20$

$x=4$

since $x+y=6 \Rightarrow 4+y=6$

$\therefore \quad y=2.$

$\therefore$ solution $(x,y)=(4,2)$

(ii) $3x-y=1$, $x-2y=-8$ $\Rightarrow$ $6x-2y=2$, $x-2y=-8$

(subtracting) $5x=10$

$x=2$

since $x-2y=-8 \Rightarrow 2-2y=-8$

$-2y=-10$

$y=5$

$\therefore$ solution $(x,y)=(2,5)$

(iii) $2x-5y=1$, $4x-3y-9=0$ $\Rightarrow$ $4x-10y=2$, $4x-3y=9$

(subtracting) $-7y=-7$

$y=1$

since $2x-5y=1 \Rightarrow 2x-5(1)=1$

$\Rightarrow \quad 2x=6$

$x=3$

$\therefore$ solution $(x,y)=(3,1)$

Q**2**. (i)

$$\begin{aligned} 4x-5y&=22 \\ 7x+3y-15&=0 \end{aligned} \quad\Rightarrow\quad \begin{aligned} 12x-15y&=66 \\ 35x+15y&=75 \end{aligned}$$

(adding) $47x=141$

$$x=\frac{141}{47}=3$$

since $4x-5y=22 \Rightarrow 4(3)-5y=22$

$$-5y=22-12$$
$$-5y=10$$
$$y=-2$$

$\therefore$ solution $(x,y)=(3,-2)$

(ii)

$$\begin{aligned} \frac{x}{2}-\frac{y}{6}&=\frac{1}{6} \\ x-2y&=-8 \end{aligned} \quad\Rightarrow\quad \begin{aligned} 3x-y&=1 \\ x-2y&=-8 \end{aligned}$$

$$\Rightarrow\quad \begin{aligned} 6x-2y&=2 \\ x-2y&=-8 \end{aligned}$$

(subtracting) $5x=10$

$$x=2$$

since $3x-y=1 \Rightarrow 3(2)-y=1$

$$6-y=1$$
$$-y=-5$$
$$y=5$$

$\therefore$ solution $(x,y)=(2,5)$

(iii)

$$\begin{aligned} \frac{4x-2}{5}&=\frac{8y}{10} \\ 18x-20y&=4 \end{aligned} \quad\Rightarrow\quad \begin{aligned} 8x-4&=8y \\ 18x-20y&=4 \end{aligned}$$

$$\Rightarrow\quad \begin{aligned} 8x-8y&=4 \\ 18x-20y&=4 \end{aligned}$$

$$\Rightarrow\quad \begin{aligned} 40x-40y&=20 \\ 36x-40y&=8 \end{aligned}$$

(subtracting) $4x=12$

$$x=3$$

since $18x-20y=4$

$$\Rightarrow\ 18(3)-20y=4$$
$$-20y=4-54=-50$$
$$y=2\tfrac{1}{2}$$

$\therefore$ solution $(x,y)=(3,2\tfrac{1}{2})$

Q**3**. $\frac{2x-5}{3}+\frac{y}{5}=6 \Rightarrow 5(2x-5)+3y=15.6$

$$\Rightarrow 10x-25+3y=90$$

$$10x+3y=115$$

$\frac{3x}{10}+2=\frac{3y-5}{2} \Rightarrow 3x+10.2=5(3y-5)$

$$\Rightarrow 3x+20=15y-25$$

$$\Rightarrow 3x-15y=-45$$

$\therefore 10x+3y=115$ $\Rightarrow$ $50x+15y=575$

$3x-15y=-45$ $\quad$ $\underline{3x-15y=-45}$

(adding) $\quad 53x=530$

$$x=10$$

since $10x+3y=115$

$$10(10)+3y=115$$

$$3y=15$$

$$y=5$$

$\therefore$ solution $(x, y)=(10,5)$

Q**4**. $y=3x-23 \Rightarrow y=3x-23$

$y=\frac{x}{2}+2 \Rightarrow 2y=x+4$

$$\Rightarrow 2y=6x-46$$

$$\underline{2y=x+4}$$

(subtracting) $\quad 0=5x-50$

$$\Rightarrow 5x=50$$

$$x=10$$

since $y=3x-23$

$\Rightarrow \quad y=3(10)-23=7$

$\therefore$ solution $(x, y)=(10,7)$

Q5. (i)

$$\begin{aligned}
&A:\ 2x+y+z=8 && 3A:\ 6x+\cancel{3y}+3z=24\\
&B:\ 5x-3y+2z=3 \quad\Rightarrow && B:\ \underline{5x-\cancel{3y}+2z=3}\\
&C:\ 7x+y+3z=20 && D:\text{(adding)}\ 11x \quad +5z=27\\
& && \text{also}\quad B:\ 5x-\cancel{3y}+2z=3\\
& && 3C:\ \underline{21x+\cancel{3y}+9z=60}\\
& && E:\text{(adding)}\ 26x \quad +11z=63
\end{aligned}$$

$$\begin{aligned}
\Rightarrow 11D:&\ 121x+\cancel{55z}=297\\
5E:&\ \underline{130x+\cancel{55z}=315}\\
\text{(subtracting)}&\quad -9x=-18\\
&\quad x=2\\
\text{since}&\quad 11x+5z=27\\
&\quad 11(2)+5z=27\\
&\quad 5z=27-22=5\\
&\quad z=1\\
\text{since}&\quad 2x+y+z=8\\
&\quad 2(2)+y+1=8\\
&\quad y=3
\end{aligned}$$

$\therefore$ solution $(x,y,z)=(2,3,1)$

(ii)

$$\begin{aligned}
&A:\ 2x-y-z=6 && 2A:\ 4x-\cancel{2y}-2z=12\\
&B:\ 3x+2y+3z=3 \quad\Rightarrow && B:\ \underline{3x+\cancel{2y}+3z=3}\\
&C:\ 4x+y-2z=3 && D:\text{(adding)}\ 7x \quad +z=15\\
& && \text{also}\quad B:\ 3x+\cancel{2y}+3z=3\\
& && 2C:\ \underline{8x+\cancel{2y}-4z=6}\\
& && E:\text{(subtracting)}\ -5x \quad +7z=-3
\end{aligned}$$

$$\begin{aligned}
\text{since } D:\ 7x+z=15 \Rightarrow 7D &= 49x+\cancel{7z}=105\\
\text{also } E &= \underline{-5x+\cancel{7z}=-3}\\
\text{subtracting:}\quad & 54x=108\\
& x=2.
\end{aligned}$$

$$\begin{aligned}
\text{since } D\ &:7x+z=15\\
\Rightarrow\quad & 7(2)+z=15\\
& z=1\\
\text{also, since } A&:2x-y-z=6\\
\Rightarrow\quad & 2(2)-y-1=6\\
& -y=3\\
& y=-3.
\end{aligned}$$

$\therefore$ solution $(x,y,z)=(2,-3,1)$

(iii) $A:\ 2x+y-z=9$ $\qquad$ $A:\ 2x+y-\not{z}=9$

$B:\ x+2y+z=6 \quad \Rightarrow \quad B:\ x+2y+\not{z}=6$

$C:\ 3x-y+2z=17 \qquad D\text{(adding)}:\ 3x+3y=15$

$\text{also } 2B:\ 2x+4y+\not{2z}=12$

$C:\ 3x-y+\not{2z}=17$

$E\text{(subtracting)}:-x+5y=-5$

$$\begin{aligned}
&\text{since } D: \quad 3x+3y=15\\
&3E:-3x+15y=-15\\
&\text{adding}: \quad 18y=0\\
&y=0.\\
&\text{since } E: \quad -x+5y=-5\\
&-x+5(0)=-5\\
&x=5.\\
&\text{also } A:\ 2x+y-z=9\\
&2(5)+0-z=9\\
&-z=9-10=-1\\
&+z=1
\end{aligned}$$

$\therefore$ solution $(x,y,z)=(5,0,1)$

Q**6**. (i) $A:\ 2a+b+c=8 \quad \Rightarrow \quad 3A:\ 6a+\not{3b}+3c=24$

$B:\ 5a-3b+2c=-3 \qquad B:\ 5a-\not{3b}+2c=-3$

$C:\ 7a-3b+3c=1 \qquad D:\text{adding}:\ 11a \quad +5c=21$

$\text{also} \quad B:\ 5a-\not{3b}+2c=-3$

$C:\ 7a-\not{3b}+3c=1$

$E:\ \text{subtracting}:\ -2a \quad -c=-4.$

$$\begin{aligned}
&\text{since } D: \quad 11a+\not{5c}=21\\
&\text{and } 5E:-10a-\not{5c}=-20\\
&\text{adding}: \quad a \quad =1\\
&\text{since } D: 11a+5c=21\\
&\Rightarrow \quad 11(1)+5c=21\\
&5c=10\\
&c=2.\\
&\text{also, } A: 2a+b+c=8\\
&2(1)+b+2=8\\
&b=4
\end{aligned}$$

$\therefore$ solution $(a,b,c)=(1,4,2)$

(ii) $A:\ x+y+2z=3 \qquad \Rightarrow \qquad 2A:\ 2x+\not{2y}+4z=6$

$B:\ 4x+2y+z=13 \qquad\qquad B:\ \underline{4x+\not{2y}+z=13}$

$C:\ 2x+y-2z=9 \qquad D:\text{(subtracting)}:\ -2x\quad +3z=-7$

also $\qquad B:\ \not{4x}+\not{2y}+z=13$

$2C:\ \underline{\not{4x}+\not{2y}-4z=18}$

$E:\text{(subtracting)}:\qquad 5z=-5$

$\Rightarrow \qquad z=-1$

since $D:\quad -2x+3z=-7$

$\Rightarrow -2x+3(-1)=-7$

$-2x-3=-7$

$-2x=-7+3$

$-2x=-4$

$x=2.$

also, $A:\quad x+y+2z=3$

$2+y+2(-1)=3$

$y=3$

$\therefore$ solution $(x,y,z)=(2,3,-1).$

(iii) $A:\ x+y+z=2 \qquad\qquad A:\ x+y+z=2$

$B:\ 2x+3y+z=7$

$C:\ \frac{x}{2}-\frac{y}{6}+\frac{z}{3}=\frac{2}{3} \qquad \Rightarrow \qquad C:\ \underline{3x-y+2z=4}$

$D:\text{(adding)}\quad 4x\quad +3z=6$

also $B:\ 2x+3y+z=7$

$\underline{3C:\ 9x-3y+6z=12}$

$E:\text{(adding)}\quad 11x\quad +7z=19$

$\therefore\ 11D:\ 44x+33z=66$

$4E:\ \underline{44x+28z=76}$

subtracting: $\qquad 5z=-10$

$z=-2.$

since $D: 4x+3z=6$

$4x+3(-2)=6$

$4x=12$

$x=3$

also, $A: x+y+z=2$

$\Rightarrow \qquad 3+y-2=2$

$y=2-1$

$y=1$

$\therefore$ solution $(x,y,z)=(3,1,-2)$

Q**7**. $A: \; 6x+4y-2z-5=0 \quad \Rightarrow \quad A: \; 6x+\cancel{4y}-2z=5$

$B: \; 3x-2y+4z+10=0 \qquad 2B: \; 6x-\cancel{4y}+8z=-20$

$C: \; 5x-2y+6z+13=0 \qquad D:\text{(adding)}: \; 12x \quad +6z=-15$

$$\text{also} \quad B: \; 3x-\cancel{2y}+4z=-10$$

$$C: \; 5x-\cancel{2y}+6z=-13$$

$$E\text{(subtracting)}: \; -2x \quad -2z=3$$

also, $D: \; 12x+\cancel{6z}=-15$

$3E: \; -6x-\cancel{6z}=9$

adding: $6x \quad =-6$

$x=-1$

since $D: \; 12x+6z=-15$

$12(-1)+6z=-15$

$6z=-3$

$z=-\tfrac{1}{2}$

also, A : $6x+4y-2z=5$

$\Rightarrow \; 6(-1)+4y-2\left(-\tfrac{1}{2}\right)=5$

$-6+4y+1=5$

$4y=10$

$y=2\tfrac{1}{2}$

$\therefore$ solution $(x,y,z)=(-1,2\tfrac{1}{2},-\tfrac{1}{2})$.

Q**8**. Curve $f(x)=ax^2+bx+c$

$(1,2)$ on curve $\Rightarrow$ when $x=1,\; f(x)=2$

$\Rightarrow 2=a(1)^2+b(1)+c$

$\Rightarrow 2=a+b+c \qquad : A$

$(2,4)$ on curve $\Rightarrow$ when $x=2,\; f(x)=4$

$\Rightarrow 4=a(2)^2+b(2)+c$

$\Rightarrow 4=4a+2b+c \qquad : B$

$(3,8)$ on curve $\Rightarrow$ when $x=3,\; f(x)=8$

$\Rightarrow 8=a(3)^2+b(3)+c$

$\Rightarrow 8=9a+3b+c \qquad : C$

$$\begin{aligned}
\text{since } A: &\quad a + b + \not{c} = 2 \\
\text{and } B: &\quad \underline{4a + 2b + \not{c} = 4} \\
D(\text{subtracting}): &\quad -3a - b \quad = -2
\end{aligned}$$

$$\begin{aligned}
\text{since } B: &\quad 4a + 2b + \not{c} = 4 \\
\text{and } C: &\quad \underline{a + 3b + \not{c} = 8} \\
E(\text{subtracting}): &\quad -5a - b \quad = -4
\end{aligned}$$

$$\begin{aligned}
\text{also} \quad D: &\quad -3a - \not{b} = -2 \\
E: &\quad \underline{-5a - \not{b} = -4} \\
\text{subtracting}: &\quad 2a \quad = 2 \\
\Rightarrow &\quad a = 1
\end{aligned}$$

$$\begin{aligned}
\text{since} \quad D: &\quad -3a - b = -2 \\
\Rightarrow &\quad -3(1) - b = -2 \\
&\quad -b = +1 \\
&\quad b = -1
\end{aligned}$$

$$\begin{aligned}
\text{also} \quad A: &\quad a + b + c = 2 \\
\Rightarrow &\quad 1 - 1 + c = 2 \\
&\quad c = 2
\end{aligned}$$

$\therefore$ solution $(a, b, c) = (1, -1, 2)$

Q**9**. point 1, $(1,1)$

point 2, $(0,-6)$

point 3, $(-2,-8)$

Curve = $f(x) = ax^2 + bx + c$

$(1,1)$ on curve $\Rightarrow$ when $x = 1,\ f(x) = 1$

$\Rightarrow 1 = a(1)^2 + b(1) + c$

$\Rightarrow 1 = a + b + c \qquad : A$

$(0,-6)$ on curve $\Rightarrow$ when $x = 0,\ f(x) = -6$

$\Rightarrow -6 = a(0) + b(0) + c$

$\Rightarrow -6 = c \qquad : B$

$(-2,-8)$ on curve $\Rightarrow$ when $x = -2,\ f(x) = -8$

$\Rightarrow -8 = a(-2)^2 + b(-2) + c$

$-8 = 4a - 2b + c \qquad : C$

$$\begin{aligned}
\text{also}\quad A: &\quad a+b-6=1\\
C: &\quad 4a-\cancel{2b}-6=-8\\
\Rightarrow\quad 2A: &\quad 2a+\cancel{2b}-12=\ \ 2\\
\text{adding}: &\quad 6a-18=-6\\
&\quad 6a=12\\
&\quad a=2
\end{aligned}$$

$$\begin{aligned}
\text{since } A: &\quad a+b-6=1\\
\Rightarrow &\quad 2+b-6=1\\
&\quad b=1+4=5
\end{aligned}$$

$(a,b,c)=(2,5,-6)$

solution: $f(x)=2x^2+5x-6$

Q**10**. Let $x=$ number of people paying € 20

Let $y=$ number of people paying € 30

$\therefore \quad x+y=44,000 \quad : A$

also, $20x+30y=1200000 : B$

$$\begin{aligned}
\therefore\quad 20A: &\quad \cancel{20}x+20y=\ \ 880000\\
B: &\quad \cancel{20}x+30y=1200000\\
\text{subtracting}: &\quad -10y=-320000\\
&\quad y=32,000
\end{aligned}$$

$\therefore$ 32,000 paid the higher price.

Q**11**. Let x be Lydia's age now.

$\therefore$ five years ago, Lydia was $(x-5)$ years old.

Let Callum be y years old now.

$\therefore$ three years from now, Callum will be $(y+3)$ years old

$\Rightarrow \ (y+3)=2(x-5) \quad : A$

also, $\dfrac{x+y}{2}=16$

$\Rightarrow \quad x+y=32 \quad : B$

$$\begin{aligned}
\text{From } A: &\quad y+3=2x-10\\
&\quad 13=2x-y\\
\therefore\quad A: &\quad 2x-y=13\\
B: &\quad x+y=32\\
\text{adding}: &\quad 3x\quad =45\\
&\quad x=15
\end{aligned}$$

since $x+y=32$

$\Rightarrow\quad 15+y=32$

$y=17$

Lydia is 15 years old, Callum is 17 years old.

Q**12**. Equation of line: $y=ax+b$.

$(6,7)$ on line $\Rightarrow$ when $x=6, y=7$

$\Rightarrow 7=a(6)+b$

$\Rightarrow 6a+b=7 \qquad :A$

also, $(-2,3)$ on line $\Rightarrow$ when $x=-2, y=3$

$\Rightarrow 3=a(-2)+b$

$\Rightarrow -2a+b=3 \qquad :B$

since $\quad A: \quad 6a+\not{b}=7$

and $\quad B: \quad -2a+\not{b}=3$

subtracting: $\quad 8a \quad =4$

$\Rightarrow \qquad a \quad =\frac{4}{8}=\frac{1}{2}.$

since $A: 6a+b=7$

$\Rightarrow 6\left(\frac{1}{2}\right)+b=7$

$b=4$

$\therefore$ line $y=\frac{1}{2}x+4.$

Verify $(4,6)$ is on line $\Rightarrow 6=\frac{1}{2}(4)+4$

$=6$, which is true.

Q**13**. $\frac{N_1}{4}-N_2=0 \quad \Rightarrow \quad N_1-4N_2=0 \qquad :A$

$N_1+\frac{1}{2}N_2-99=0 \quad \Rightarrow \quad 2N_1+N_2=198 \quad :B$

since $\quad A: \quad N_1-4N_2=0$

and $\quad 4B: \quad 8N_1+4N_2=792$

adding: $\quad 9N_1 \quad =792$

$N_1 \quad =88$

also, $A: \quad N_1-4N_2=0$

$\Rightarrow \quad 88-4N_2=0$

$-4N_2=-88$

$N_2=22$

$(N_1,N_2)=(88,22)$

Q14. $$\frac{a}{x-2}+\frac{b}{x+2}=\frac{4}{(x-2)(x+2)}$$

$$\Rightarrow \frac{a(x+2)+b(x-2)}{(x-2)(x+2)}=\frac{4}{(x-2)(x+2)}$$

$$\Rightarrow \quad ax+2a+bx-2b=4$$

$$\Rightarrow (a+b)x+2a-2b=4+0.x$$

$$\Rightarrow \quad a+\ b=0 \quad :A$$

and $\quad 2a-2b=4 \quad :B$

also $\quad \underline{2a+2b=0} \quad :2A$

adding: $\quad 4a \quad =4$

$$a \quad =1$$

since $\quad a+b=0$

$$\Rightarrow \quad 1+b=0$$

$$b=-1$$

$$\therefore \quad (a,b)=(1,-1)$$

$$\therefore \quad \frac{1}{x-2}+\frac{-1}{x+2}=\frac{4}{(x-2)(x+2)}$$

$$\frac{x+2-(x-2)}{(x-2)(x+2)}=\frac{4}{(x-2)(x+2)}$$

$$\frac{\not{x}+2-\not{x}+2}{(x-2)(x+2)}=$$

$$\frac{4}{(x-2)(x+2)}=\frac{4}{(x-2)(x+2)} \qquad \text{qed}$$

Q15. $$\frac{c}{z-3}+\frac{d}{z+2}=\frac{4}{(z-3)(z+2)}$$

$$\Rightarrow \frac{c(z+2)+d(z-3)}{(z-3)(z+2)}=\frac{4}{(z-3)(z+2)}$$

$$\Rightarrow \quad cz+2c+dz-3d=4$$

$$(c+d)z+2c-3d=0.z+4$$

$$\therefore \quad c+d=0 \quad :A$$

and $\quad 2c-3d=4 \quad :B$

$$\Rightarrow \quad \underline{2c+2d=0} \quad :2A$$

subtracting: $\quad -5d=4$

$$d=\frac{-4}{5}$$

since $\quad c+d=0$

$$\Rightarrow \quad c-\frac{4}{5}=0$$

$$\Rightarrow \quad c=\frac{4}{5}$$

$$\therefore \quad (c,d)=\left(\frac{4}{5},-\frac{4}{5}\right)$$

$$\therefore \quad \frac{4}{5(z-3)} + \frac{-4}{5(z+2)} = \frac{4}{(z-3)(z+2)}$$

$$\Rightarrow \frac{4(z+2)-4(z-3)}{5(z-3)(z+2)} =$$

$$\frac{\cancel{4z}+8-\cancel{4z}+12}{5(z-3)(z+2)}$$

$$\frac{20}{5(z-3)(z+2)}$$

$$= \frac{4}{(z-3)(z+2)} = \frac{4}{(z-3)(z+2)} \qquad \text{qed}$$

Q**16**. Let x = number of litres of 70% alcohol

$\Rightarrow x(70\%) + 50(40\%) = (x+50)(50\%)$

$$\Rightarrow \quad 0.7x + 0.4(50) = 0.5x + 0.5(50)$$

$$0.7x + 20 = 0.5x + 25$$

$$0.7x - 0.5x = 25 - 20$$

$$0.2x = 5$$

$$x = 25 \text{ litres.}$$

Q**17**. x = bigger number

y = smaller number

$A: \quad x + \quad y = 26$

$B: \quad 4x - 5y = 5$

$\Rightarrow 5A: \quad 5x + 5y = 130$

$B: \quad 4x - 5y = \quad 5$

adding: $9x \qquad = 135$

$x \qquad = 15$

since $\qquad x + y = 26$

$\Rightarrow \qquad 15 + y = 26$

$y = 11$

$\therefore \quad (x, y) = (15, 11)$

Q**18**. $v = u + at \qquad v$ = speed

t = time.

at $t = 7, v = 2 \quad \Rightarrow \quad 2 = u + 7a \quad : A$

at $t = 13, v = 5 \quad \Rightarrow \quad 5 = u + 13a \quad : B$

since $A: u + 7a = 2$

$B: \underline{u + 13a = 5}$

subtracting: $-6a = -3$

$6a = 3$

$a = \frac{3}{6} = \frac{1}{2}$

since $u + 7a = 2$

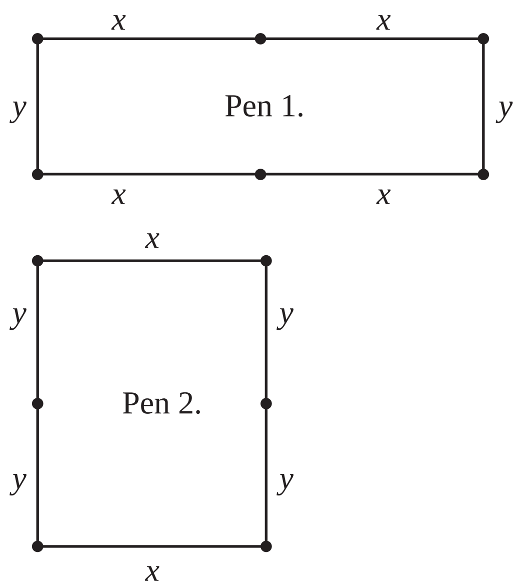
$\Rightarrow u + 7\left(\frac{1}{2}\right) = 2$

$u = 2 - 3\frac{1}{2}$

$u = -1\frac{1}{2}$

Q19. $\therefore \quad 4x + 2y = 60 \quad : A$

also, $2x + 4y = 42 \quad : B$



$\therefore \quad 4x + 2y = 60 \quad : A$

also $\underline{4x + 8y = 84} \quad : 2B$

subtracting: $-6y = -24$

$y = 4$

since $4x + 2y = 60$

$\Rightarrow 4x + 2(4) = 60$

$4x = 52$

$x = 13$

$\therefore$ Original pen dimensions: 26×4

new pen dimensions: 13×8

Pen 1 requires $4x$ and $2y$ lengths

Pen 2 requires $2x$ and $4y$ lengths

$\Rightarrow$ they both have $2x$ and $2y$ lengths in common

$\therefore$ in pen 2, $2x$ lengths are swapped for $2y$ lengths

if $y < x$, less fencing is needed in pen 2.

Note: Area of pen $1 = (2x) \times (y) = 2xy$

Area of pen $2 = (x) \times (2y) = 2xy$

$\Rightarrow$ the areas are the same.

Q**20**. $y = ax^2 + bx + c$

the point $(0,1) \in$ curve $\Rightarrow$ when $x = 0, y = 1$

$$\Rightarrow 1 = a(0)^2 + b(0) + c$$
$$\Rightarrow 1 = c$$

the point $(2,9) \in$ curve $\Rightarrow$ when $x = 2, y = 9$

$$\Rightarrow 9 = a(2)^2 + b(2) + c$$
$$\Rightarrow 9 = 4a + 2b + c$$
$$\Rightarrow 9 = 4a + 2b + 1 \quad \text{since } c = 1$$
$$\Rightarrow 8 = 4a + 2b$$
$$A: \quad \Rightarrow 4 = 2a + b \quad \text{dividing each term by 2.}$$

the point $(4,41) \in$ curve $\Rightarrow$ when $x = 4, y = 41$

$$\Rightarrow 41 = a(4)^2 + b(4) + c$$
$$\Rightarrow 41 = 16a + 4b + c$$
$$\Rightarrow 41 = 16a + 4b + 1 \quad \text{since } c = 1$$
$$\Rightarrow 40 = 16a + 4b$$
$$B: \quad \Rightarrow 10 = 4a + b \quad \text{dividing each term by 4.}$$

since $A: \; 2a + \not{b} = 4$

and $B: \; \underline{4a + \not{b} = 10}$

subtracting: $-2a = -6$

$a = 3$

since $A: \; 2a + b = 4$

$\Rightarrow \; 2(3) + b = 4$

$b = -2$

$\therefore \; (a,b,c) = (3,-2,1)$

Q**21**. (i) $A: \; y - z = 3$

$B: \; x - 2y + z = -4$

$C: \; x + 2y = 11$

$\therefore \quad A: \; y - \not{z} = 3$

$B: \; \underline{x - 2y + \not{z} = -4}$

D(adding): $\not{x} - y = -1$

C : $\underline{\not{x} + 2y = 11}$

subtracting: $-3y = -12$

$y = 4$

since $x - y = -1$

$\Rightarrow \quad x - 4 = -1$

$x = 3$

also $\quad y - z = 3$

$\Rightarrow \quad 4 - z = 3$

$-z = -1$

$z = 1$

solution $(x, y, z) = (3, 4, 1)$

(ii) $A: \ \frac{x}{3} + \frac{y}{2} - z = 7 \Rightarrow 2x + 3y - 6z = 42$

$B: \ \frac{x}{4} - \frac{3y}{2} + \frac{z}{2} = -6 \Rightarrow x - 6y + 2z = -24$

$C: \ \frac{x}{6} - \frac{y}{4} - \frac{z}{3} = 1 \Rightarrow 2x - 3y - 4z = 12$

$\therefore \quad A: \ \cancel{2x} + \ 3y - 6z = 42$

$2B: \ \cancel{2x} - 12y + 4z = -48$

$D\text{(subtracting)}: \quad 15y - 10z = 90$

also $\quad 2B: \ \cancel{2x} - 12y + 4z = -48$

$C: \ \cancel{2x} - \ 3y - 4z = 12$

$E\text{(subtracting)}: \quad -9y + 8z = -60$

$\therefore \quad 4D: \ 60y - \cancel{40z} = 360$

$5E: \ -45y + \cancel{40z} = -300$

adding: $\quad 15y = 60$

$y = 4$

since $\quad D: \ 15 - 10z = 90$

$15(4) - 10z = 90$

$-10z = 30$

$z = -3$

also $\quad A: \ 2x + 3y - 6z = 42$

$2x + 3(4) - 6(-3) = 42$

$2x + 12 + 18 = 42$

$2x = 12$

$x = 6$

$\therefore \quad (x, y, z) = (6, 4, -3)$

Q**22**. curve : $x^2+y^2+ax+by+c=0.$

$(1,0)\in$ curve $\Rightarrow$ when $x=1, y=0$

$$\Rightarrow 1^2+0^2+a(1)+b(0)+c=0$$
$$\Rightarrow 1+a+c=0$$
$$\Rightarrow a+c=-1 \qquad :A$$

$(1,2)\in$ curve $\Rightarrow$ when $x=1, y=2$

$$\Rightarrow 1^2+2^2+a(1)+b(2)+c=0$$
$$\Rightarrow 1+4+a+2b+c=0$$
$$\Rightarrow a+2b+c=-5 \qquad :B$$

$(2,1)\in$ curve $\Rightarrow$ when $x=2, y=1$

$$\Rightarrow 2^2+1^2+a(2)+b(1)+c=0$$
$$\Rightarrow 4+1+2a+b+c=0$$
$$\Rightarrow 2a+b+c=-5 \qquad :C$$

since $B:\quad a+\cancel{2b}+\ c=-5$

and $2C:\quad 4a+\cancel{2b}+2c=-10$

D(subtracting) : $-3a \quad -\cancel{c}=5$

$A:\quad a \quad +\cancel{c}=-1$

adding : $-2a \quad =4$

$a \quad =-2$

since $A:\quad a+c=-1$

$\Rightarrow\quad -2+c=-1$

$c=1$

also $B:\quad a+2b+c=-5$

$-2+2b+1=-5$

$2b=-4$

$b=-2$

$\therefore\ (a,b,c)=(-2,-2,1)$

Revision Exercise (Core)

Q**1**. (i) $\dfrac{12m^2n^3}{(6m^4n^5)^2} = \dfrac{\overset{1}{\cancel{12}}m^2n^3}{\underset{3}{\cancel{36}}\,m^8n^{10}} = \dfrac{1}{3m^6n^7}$

(ii) $\dfrac{3+\frac{1}{x}}{\frac{5}{x}+4} = \dfrac{\left(\frac{3x+1}{x}\right)}{\left(\frac{5+4x}{x}\right)} = \left(\dfrac{3x+1}{x}\right)\cdot\left(\dfrac{x}{5+4x}\right) = \dfrac{3x+1}{5+4x}$

(iii) $\dfrac{2+\frac{x}{2}}{x^2-16} = \dfrac{\frac{4+x}{2}}{(x-4)(x+4)} = \dfrac{\cancel{(x+4)}}{2(x-4)\cancel{(x+4)}} = \dfrac{1}{2x-8}$

Q**2**. (i)

$$\begin{aligned} y &= x+4 \\ 5y+2x &= 6 \end{aligned} \quad\Rightarrow\quad \begin{aligned} y-x &= 4 \quad : A \\ 5y+\cancel{2x} &= 6 \quad : B \end{aligned}$$

$\therefore \quad 2y-\cancel{2x} = 8 \quad : 2A$

adding $7y = 14$

$y = 2$

since $y = x+4$

$\Rightarrow \quad 2 = x+4$

$\Rightarrow \quad x = -2$

$\therefore \quad (x, y) = (-2, 2)$

(ii) $3x+y=7 \quad \Rightarrow \quad y = 7-3x$

$x^2+y^2=13 \quad \Rightarrow \quad x^2+(7-3x)^2 = 13$

$\Rightarrow \quad x^2+[49-42x+9x^2] = 13$

$x^2+49-42x+9x^2 = 13$

$10x^2-42x+36 = 0$

$5x^2-21x+18 = 0$

$(5x-6)(x-3) = 0$

$\therefore \quad x = 3 \quad$ or $\quad x = \dfrac{6}{5}$

$\Rightarrow \quad y = 7-3(3) \quad$ or $\quad y = 7-3\left(\dfrac{6}{5}\right)$

$y = 7-9 \quad$ or $\quad y = 7-\dfrac{18}{5}$

$y = -2 \quad$ or $\quad y = \dfrac{17}{5}$

$\therefore \quad (x, y) = (3, -2) \quad$ or $\quad \left(\dfrac{6}{5}, \dfrac{17}{5}\right)$

Q3.

$$\begin{array}{r|l} & x^2+2x-1 \\ \hline x-3 & x^3-x^2-7x+3 \\ & \underline{x^3-3x^2} \quad \text{(subtracting)} \\ & \quad 2x^2-7x+3 \\ & \quad \underline{2x^2-6x} \quad \text{(subtracting)} \\ & \qquad -x+3 \\ & \qquad \underline{-x+3} \quad \text{(subtracting)} \\ & \qquad\quad 0 \end{array}$$

answer: x^2+2x-1

Q4.

$$\begin{array}{r|l} & 3x^3+6x^2+3x+33 \\ \hline x-2 & 3x^4 \qquad -9x^2+27x-66 \\ & \underline{3x^4-6x^3} \quad \text{(subtracting)} \\ & \quad 6x^3-9x^2+27x-66 \\ & \quad \underline{6x^3-12x^2} \quad \text{(subtracting)} \\ & \qquad 3x^2+27x-66 \\ & \qquad \underline{3x^2-6x} \quad \text{(subtracting)} \\ & \qquad\quad 33x-66 \\ & \qquad\quad \underline{33x-66} \quad \text{(subtracting)} \\ & \qquad\qquad 0 \end{array}$$

answer: $3x^3+6x^2+3x+33$

Q5. (i) $x^4-9x^2=0$

$\Rightarrow x^2(x^2-9)=0$

$\Rightarrow x^2(x-3)(x+3)=0$

$\therefore\ x=0,\ 3,\ -3$

(ii) $(2x-1)^3(2-x)=0$

$\Rightarrow \quad (2x-1)^3=0$

$\Rightarrow \quad (2x-1)=0$

$x=\frac{1}{2}$

or $\quad 2-x=0$

$x=2$

$\therefore\ x=2,\ \frac{1}{2}.$

Q6. $4x^2+20x+k$ is a perfect square

$\Rightarrow (2x+a)(2x+a)=4x^2+20x+k$

$\Rightarrow \quad 4x^2+4ax+a^2=4x^2+20x+k$

$\Rightarrow \quad 4a=20$

$a=5$

$\therefore \quad a^2=5^2=25=k$

$\therefore \quad k=25\left(5^2\right)$

Q7. (i) $\left(3-\sqrt{2}\right)^2=a-b\sqrt{2}$

$\Rightarrow 9-6\sqrt{2}+\left(\sqrt{2}\right)^2=a-b\sqrt{2}$

$\Rightarrow 9-6\sqrt{2}+2$

$\Rightarrow 11-6\sqrt{2}=a-b\sqrt{2}$

$\therefore \quad a=11,\ b=6$

(ii) $\dfrac{1-\sqrt{2}}{1+\sqrt{2}}=a\sqrt{2}-b$

$\Rightarrow \quad \dfrac{1-\sqrt{2}}{1+\sqrt{2}}\times\dfrac{\left(1-\sqrt{2}\right)}{\left(1-\sqrt{2}\right)}=$

$\Rightarrow \quad \dfrac{1-\sqrt{2}-\sqrt{2}+2}{1-\cancel{\sqrt{2}}+\cancel{\sqrt{2}}-2}=$

$\Rightarrow \quad \dfrac{3-2\sqrt{2}}{-1}=$

$\Rightarrow \quad -3+2\sqrt{2}=a\sqrt{2}-b$

$\therefore \quad a=2,\ b=3$

Q8. $x^3-27=x^3-3^3=(x-3)(x^2+3x+9)$

Q9. $p(x-q)^2+r=2x^2-12x+5$ for all values of x

$\Rightarrow p(x^2-2xq+q^2)+r=$

$\Rightarrow px^2-2pqx+pq^2+r=2x^2-12x+5.$

$\therefore p=2,\ -2pq=-12$ and $pq^2+r=5.$

$\Rightarrow \quad -2(2)q=-12$

$\Rightarrow \quad q=3$ and $2(3)^2+r=5$

$r=5-18$

$r=-13$

$(p,q,r)=(2,3,-13)$

Q**10**. $A: \; 3x+5y-z=-3$

$B: \; 2x+y-3z=-9$

$C: \; x+3y+2z=7$

$$\Rightarrow 2A: \; 6x+10y-\cancel{2z}=-6$$

$$C: \; x+3y+\cancel{2z}=7$$

$$D(\text{adding}): \; 7x+13y=1$$

also $$3A: \; 9x+15y-\cancel{3z}=-9$$

$$B: \; 2x+y-\cancel{3z}=-9$$

$$E(\text{subtracting}): \; 7x+14y=0$$

$\therefore$ $$D: \; 7x+13y=1$$

$$E: \; 7x+14y=0$$

(subtracting): $$-y=1$$

$$y=-1$$

since $$7x+14y=0$$

$$7x+14(-1)=0$$

$$7x=14$$

$$x=2.$$

also, $C:$ $$x+3y+2z=7$$

$$2+3(-1)+2z=7$$

$$2z=8$$

$$z=4.$$

$\therefore \; (x, y, z)=(2, -1, 4)$

Q**11**. $(b+1)^3-(b-1)^3$

$=b^3+3b^2+3b+1-(b^3-3b^2+3b-1)$

$=\cancel{b^3}+3b^2+\cancel{3b}+1-\cancel{b^3}+3b^2-\cancel{3b}+1$

$=6b^2+2.$

Q**12**. (i) 3, 12, 27, 48, 75 …

first difference: 9, 15, 21, 27 …

second difference: 6, 6, 6 … $\Rightarrow$ quadratic pattern of the form $ax^2 + bx + c$.

$\Rightarrow \quad 2a = 6$

$a = 3.$

$\therefore \quad 3x^2 + bx + c$ represents the pattern.

let $\quad x = 1 \Rightarrow 3(1)^2 + b(1) + c = 3$

$b + c = 0 \quad : \quad A$

let $\quad x = 2 \Rightarrow 3(2)^2 + b(2) + c = 12$

$2b + c = 0 \quad : \quad B$

$\Rightarrow \qquad A: \quad b + c = 0$

$B: \quad \underline{2b + c = 0}$

(subtracting) : $\quad -b \quad = 0$

$\Rightarrow \quad b \quad = 0$

since $\quad b + c = 0$

$\Rightarrow 0 + c = 0$

$\Rightarrow \quad c = 0$

$\therefore \quad ax^2 + bx + c = 3x^2.$

(ii) 5, 20, 45, 80, 125 …

first difference: 15, 25, 35, 45 …

second difference: 10, 10, 10, …

$\therefore$ quadratic pattern of the form $ax^2 + bx + c$.

$\Rightarrow 2a = 10$

$a = 5 \qquad \therefore \; 5x^2 + bx + c$ represents the pattern.

We note that for $x = 1, \quad 5x^2 = 5$

$x = 2, \quad 5x^2 = 20$

$x = 3, \quad 5x^2 = 45$ etc.

$\Rightarrow b$ and c must equal zero.

[alternatively, set up simultaneous equations in b and c and solve]

$\therefore$ the quadratic pattern is $5x^2$.

(iii) $0.5, 2, 4.5, 8, 12.5, \ldots$

first difference $= 1.5, 2.5, 3.5, 4.5, \ldots$

second difference = 1, 1, 1

$\therefore$ quadratic pattern of the form $ax^2 + bx + c$.

$\Rightarrow 2a = 1$

$a = ½$ $\therefore$ $0.5x^2 + bx + c$ represents this pattern.

By comparison to part (ii), we can deduce that the quadratic pattern is $0.5x^2$.

Q**13**. $6, 12, 20, 30, 42, \ldots$

first difference: $6, 8, 10, 12, \ldots$

second difference: $2, 2, 2, \ldots$

$\therefore$ quadratic pattern of the form $ax^2 + bx + c$.

$\Rightarrow 2a = 2$

$a = 1 \quad \Rightarrow \quad x^2 + bx + c$ represents this pattern.

let $x = 1 \quad \Rightarrow \quad 1^2 + b(1) + c = 6$

$b + c = 5 \quad : A$

let $x = 2 \quad \Rightarrow \quad 2^2 + b(2) + c = 12$

$2b + c = 8 \quad : B$

since $A: \quad b + c = 5$

and $B: \quad \underline{2b + c = 8}$

subtracting: $-b = -3$

$b = 3$

also $A: \quad b + c = 5$

$\Rightarrow \quad 3 + c = 5$

$c = 2.$

$\therefore$ quadratic pattern is $x^2 + 3x + 2$.

When $x = 100 \quad \Rightarrow \quad 100^2 + 3(100) + 2 = 10{,}302.$

Q**14**. Let the width $= x$ cm.

Let the length $= y$ cm.

$\Rightarrow \ A: \quad 3x = 2y + 3$

$B: \quad 4y = 2(x + y) + 12$

$\Rightarrow \ B: \quad 4y = 2x + 2y + 12$

$\Rightarrow \ B: \quad 2y = 2x + 12.$

since $A: \; 3x - \cancel{2y} = 3$

and $B: \; 2x - \cancel{2y} = -12$

(subtracting): $x = 15 \quad \Rightarrow \quad \text{width} = 15\,\text{cm}.$

also $B: \; 2x - 2y = -12$

$$2(15) - 2y = -12$$
$$-2y = -42$$
$$y = 21 \quad \Rightarrow \quad \text{length} = 21\,\text{cm}.$$

Q15.

$$A: \; \frac{1}{u} + \frac{1}{v} = \frac{2}{r}$$

$$B: \; m = \frac{v - r}{r - u}$$

since $A: \; \frac{1}{u} + \frac{1}{v} = \frac{2}{r}$

$$\Rightarrow \frac{v+u}{uv} = \frac{2}{r}$$

$$\Rightarrow \frac{2}{r} = \frac{v+u}{uv}$$

$$\Rightarrow \frac{r}{2} = \frac{u.v}{v+u}$$

$$\Rightarrow r = \frac{2uv}{v+u}$$

also, $m = \frac{v-r}{r-u} = \frac{v - \frac{2uv}{v+u}}{\frac{2uv}{v+u} - u}$

$$= \frac{\left(\frac{v(v+u) - 2uv}{v+u}\right)}{\left(\frac{2uv - u(v+u)}{v+u}\right)}$$

$$= \frac{v^2 + vu - 2uv}{\cancel{(v+u)}} \cdot \frac{\cancel{(v+u)}}{2uv - uv - u^2}$$

$$= \frac{v^2 - uv}{uv - u^2} = \frac{v\,\cancel{(v-u)}}{u\,\cancel{(v-u)}} = \frac{v}{u}$$

Revision Exercise (Advanced)

Q**1**. $1, 3, 6, 10, \ldots$

first difference: $2, 3, 4, \ldots.$

second difference: $1, 1, \ldots.$

$\Rightarrow$ quadratic pattern of the form $ax^2 + bx + c$

$\Rightarrow 2a = 1$

$a = \frac{1}{2} \quad \Rightarrow \quad \frac{1}{2}x^2 + bx + c$ represents this pattern.

let $x = 1$: $\frac{1}{2}(1)^2 + b(1) + c = 1$

$\frac{1}{2} + b + c = 1$

$b + c = \frac{1}{2} \quad : A$

let $x = 2$: $\frac{1}{2}(2)^2 + b(2) + c = 3$

$2 + 2b + c = 3$

$2b + c = 1 \quad : B$

since $A: \quad b + \not{c} = \frac{1}{2}$

and $B: \quad 2b + \not{c} = 1$

(subtracting) : $-b = -\frac{1}{2}$

$b = \frac{1}{2}$

also $A: \quad b + c = \frac{1}{2}$

$\Rightarrow \quad \frac{1}{2} + c = \frac{1}{2}$

$\Rightarrow \quad c = 0$

$\therefore$ the quadratic pattern is $\frac{1}{2}x^2 + \frac{1}{2}x$

Q**2**. If x m^3 of soil is needed,

$\Rightarrow x(55\%) + 1(25\%) = (x+1)(35\%)$

$\Rightarrow \quad 0.55x + 0.25 = 0.35x + 0.35$

$\Rightarrow \quad 0.55x - 0.35x = 0.35 - 0.25$

$0.2x = 0.1$

$x = 0.5$ m^3.

Q**3**. (i) Let x kg of alloy 1 be added to

y kg of alloy 2.

$\Rightarrow x + y = 8.4 \qquad : A$

alloy 1 contains 60% gold

$\Rightarrow$ the amount of gold in the new alloy $= x(60\%) = 0.6x$

alloy 2 contains 40% gold

$\Rightarrow$ the amount of gold in the new alloy $= y(40\%) = 0.4y$

also, the total amount of gold in the new alloy $= (x+y)(50\%)$

$$= 0.5(x+y)$$

$\Rightarrow 0.6x + 0.4y = 0.5(x+y) \qquad : B$

since $\quad A: \quad x + y = 8.4$

and $\quad B: \quad 6x + 4y = 5(x+y)$

$\Rightarrow \quad B: \quad x - \not{y} = 0$ (simplifying B)

and $\quad A: \quad x + \not{y} = 8.4$

$\Rightarrow \qquad 2x = 8.4$

$\qquad x = 4.2$ kg

since $\quad B: \quad x - y = 0$

$\Rightarrow \quad 4.2 - y = 0$

$\Rightarrow \quad y = 4.2$ kg also.

Q4. $\quad (3p - 2t)x + r - 4t^2 = 0 \quad$ for all x.

$\Rightarrow (3p - 2t)x + r - 4t^2 = 0.x + 0$

$\Rightarrow \; 3p - 2t = 0 \qquad \Rightarrow t = \dfrac{3p}{2}$

and $\quad r - 4t^2 = 0.$

$\Rightarrow r - 4\left(\dfrac{3p}{2}\right)^2 = 0$

$\Rightarrow \; r - \dfrac{\not{4}.9p^2}{\not{4}} = 0$

$\Rightarrow \qquad r = 9p^2.$

Q5. $\quad \dfrac{x + y^2}{x^2} + \dfrac{x-1}{x} = -1$

$\Rightarrow x + y^2 + x(x-1) = -x^2 \quad$ [multiplying each term by x^2]

$\Rightarrow x + y^2 + x^2 - x = -x^2$

$\Rightarrow \not{x} + y^2 + x^2 - \not{x} + x^2 = 0$

$\qquad 2x^2 + y^2 = 0.$

also, $\dfrac{2x^2}{y^2} + 1 = 0.$

$\qquad \dfrac{2x^2}{y^2} = -1$

$\qquad \dfrac{x^2}{y^2} = -\dfrac{1}{2}$

Q**6**. Let the students take x litres of 10% solution

and y litres of 30% solution.

$\Rightarrow \quad x+y=10$ litres

also, $x(10\%)+y(30\%)=(x+y)15\%$

$\Rightarrow \quad 10x+30y=15(x+y)$ [multiplying each term by 100]

$\therefore \quad x+y=10 \quad : A$

and $10x+30y=15x+15y$

$\Rightarrow \; -5x+15y=0 \quad : B$

since $A: \quad x+y=10$

and $B: -5x+15y=0$

$\Rightarrow \; 5A: \quad 5x+5y=50$

adding: $\quad 20y=50$

$$y=\frac{50}{20}=2\tfrac{1}{2} \text{ litres.}$$

also, since $\quad x+y=10$

$$\Rightarrow \quad x+2\tfrac{1}{2}=10$$

$$x=7\tfrac{1}{2} \text{ litres.}$$

(i) $7\tfrac{1}{2}$ litres of 10% mixed with (ii) $2\tfrac{1}{2}$ litres of 30%.

Revision Exercise (Extended-Response Questions)

Q**1**. (a) Let x be the number of adults

y be the number of children.

$\therefore \quad x + y = 548 \qquad : A$

also, $5x + 2.5y = 2460 \quad : B$

$$\begin{aligned}
&\text{since} & A: &\quad x + y = 548 \\
&\text{and} & B: &\quad \cancel{5x} + 2.5y = 2460 \\
&\Rightarrow & 5A: &\quad \underline{\cancel{5x} + 5y = 2740} \\
&\text{(subtracting):} & &\quad -2.5y = -280 \\
& & &\quad 2.5y = 280 \\
& & &\quad y = \frac{280}{2.5} = 112.
\end{aligned}$$

since $A: \; x + y = 548$

$\Rightarrow \; x + 112 = 548$

$\Rightarrow \; x = 548 - 112$

$x = 436$

$\Rightarrow$ (i) Number of adult tickets $(x) = 436$

(ii) Number of children tickets $(y) = 112$.

(iii) Proportion of adult tickets sold $= \frac{436}{548} = 0.7956.$

(b) attendance (predicted) $= 13{,}000$

$\Rightarrow$ adults $= 0.7956 \times 13{,}000 = 10{,}343$

$\Rightarrow$ children $= 13{,}000 - 10{,}343 = 2657$

Revenue $= 10{,}343 \times (€5) + 2657 \times (€2.5) = €58{,}357.50$

Q**2**. (i) x standard sofas

y deluxe sofas

Standard sofas require 2 hours of work $\Rightarrow 2x =$ time for x sofas

Deluxe sofas require 2.5 hours of work $\Rightarrow 2.5y =$ time for y sofas

$\Rightarrow$ with 48 hours of manufacturing time: $\; 2x + 2.5y = 48 \quad : A$

(ii) also, standard sofas need 1 hour finishing $\Rightarrow x =$ time for x sofas

deluxe sofas need 1.5 hours finishing $\Rightarrow 1.5y =$ time for y sofas

$\Rightarrow$ with 26 hours of finishing time : $x + 1.5y = 26$: B

since A: $2x + 2.5y = 48$

and B: $x + 1.5y = 26$

$\Rightarrow$ $2B$: $\not{2x} + 3y = 52$

and A: $\not{2x} + 2.5y = 48$

(subtracting) : $0.5y = 4$

$y = 8$

also, since B: $x + 1.5y = 26$

$x + 1.5(8) = 26$

$x + 12 = 26$

$x = 14$

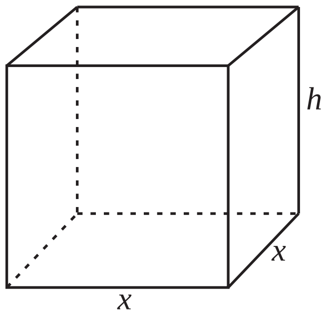
(iii) 14 standard sofas and 8 deluxe sofas.

Q3. Rectangular box with square base of length x cm.

Height of box $= h$ cm.

Volume of box $= L \cdot B \cdot H$

$= x \cdot x \cdot h = x^2 h.$



(i) If $V = 40 \Rightarrow 40 = x^2 h$

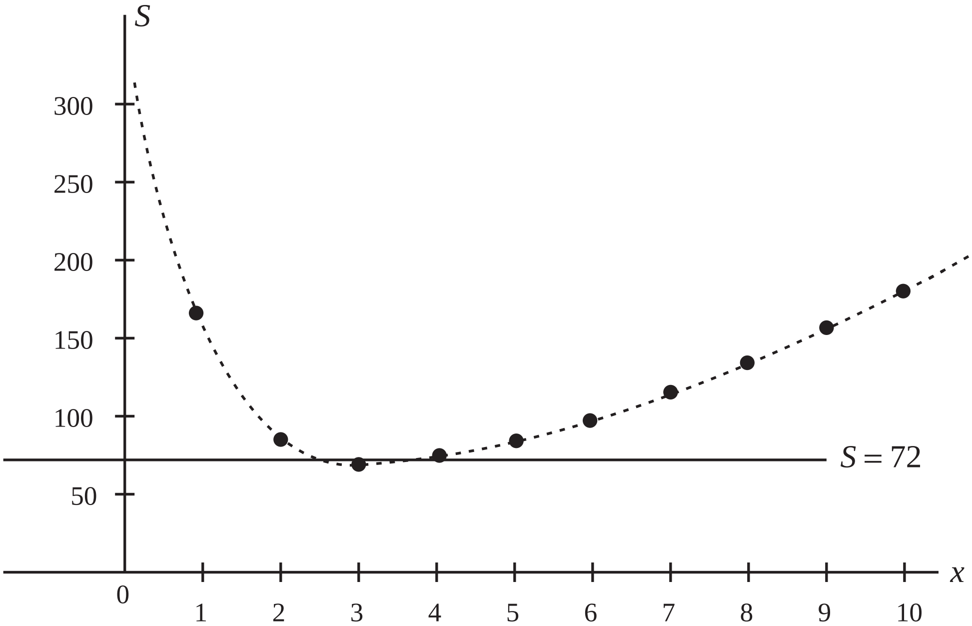
$$\Rightarrow h = \frac{40}{x^2}.$$

(ii) Surface Area $= 4 \times (x \times h) + 2x^2$

$$= 4\not{x} \cdot \frac{40}{x^{\not{2}}} + 2x^2$$

$$S = \frac{160}{x} + 2x^2.$$

(iii)



(iv) $72 = \frac{160}{x} + 2x^2$

$72x = 160 + 2x^3$

$\Rightarrow 2x^3 - 72x + 160 = 0.$

Using trail + error at $x = 2$: $2(2)^3 - 72(2) + 160 = 32 > 0$

at $x = 2.8$: $2(2.8)^3 - 72(2.8) + 160 = 2.3 > 0$

at $x = 2.9$: $2(2.9)^3 - 72(2.9) + 160 = -0.02 < 0$

$\therefore$ at $x = 2.9$ cm (approximately), $S = 72$ cm^2

also, at $x = 4$: $2(4)^3 - 72(4) + 160 = 0$

$\therefore$ at $x = 4$ cm, $S = 72$ cm^2

when $x = 4$, $h = \frac{40}{(4)^2} = 2.5$ cm.

when $x = 2.9$, $h = \frac{40}{(2.9)^2} = 4.76$ cm.

Q4. Selling price of game = €11.50.

Production cost for each game = €10.50.

Initial production costs = €3500.

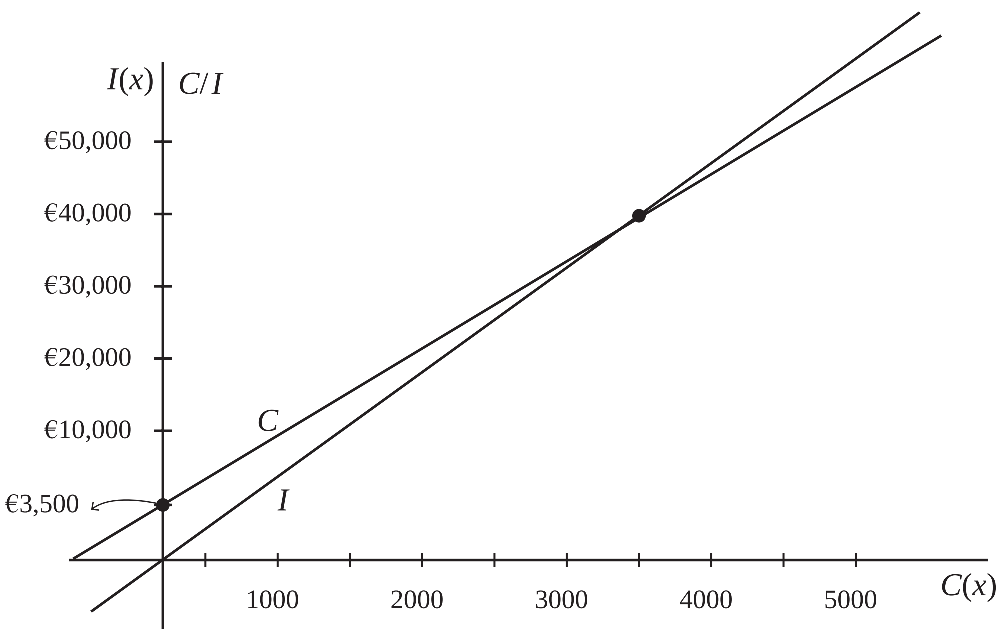
(i) Cost of producing x games $= C(x)$

$\Rightarrow C(x) = 10.5x + 3500$

(ii) Income $= I(x)$

$\Rightarrow I(x) = 11.5x.$

(iii)



(iv) To recoup costs : $11.5x = 10.5x + 3500$

$\Rightarrow$ $x = 3500$ need to be sold.

(v) $P = I - C$ $\Rightarrow$ $P =$ profit.

(vi) To make a profit of €2000:

$$\Rightarrow \quad P = I - C$$
$$\therefore \quad 2000 = 11.5x - [10.5x + 3500]$$
$$2000 = 11.5x - 10.5x - 3500$$
$$5500 = x$$

$\therefore$ 5500 need to be sold.

Q**5**. 15 days to complete quilt.

x blue squares at a rate of 4 squares a day.

y white squares at a rate of 7 squares a day.

96 squares in quilt $\Rightarrow x + y = 96 \quad : A$

15 days to finish $\Rightarrow \frac{x}{4} + \frac{y}{7} = 15 \quad : B$

$\therefore \quad A: \quad x + y = 96$

$28B: \quad 7x + 4y = 420$

also $4A: \quad 4x + 4y = 384$

(subtracting): $3x = 36$

$x = 12$

since $A: \quad x + y = 96$

$\Rightarrow \quad 12 + y = 96$

$y = 84$

(a) $\text{Cost} = x(0.8) + y(1.20)$

$= 12(0.8) + 84(1.20)$

$= €110.40$

(b) $L : W = 3x : 2x$

$\Rightarrow 3x.2x = 96$

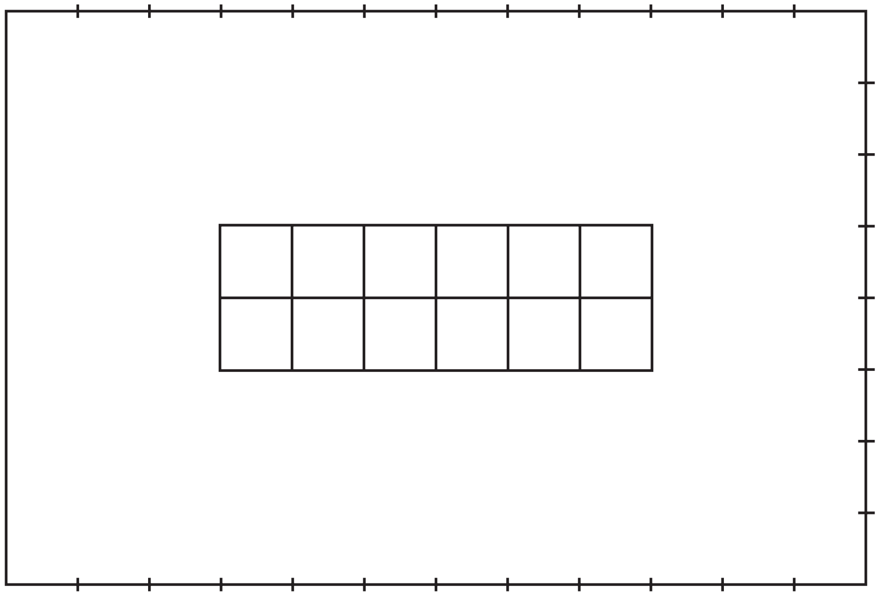
$6x^2 = 96$

$x^2 = 16$

$x = 4$

$\therefore \quad L = 3 \times 4 = 12$

$W = 2 \times 4 = 8$



The 12 blue squares could form 2 rows of 6 in the centre. (There are many different possibilities.)

Q6. overheads = €30,000 per year

cost of manufacture = €40 per wheelbarrow

(i) $C(x) = 40x + 30{,}000$

(ii) 6000 wheelbarrows per year $\Rightarrow \dfrac{30000}{6000} = €5$ overhead per wheelbarrow

$\Rightarrow$ Total cost per wheelbarrow = €40 + €5 = €45

(iii) To get a cost of €46 per wheelbarrow:

$$\frac{30{,}000}{x} + 40 = 46.$$

$$\Rightarrow \quad 30{,}000 + 40x = 46x$$

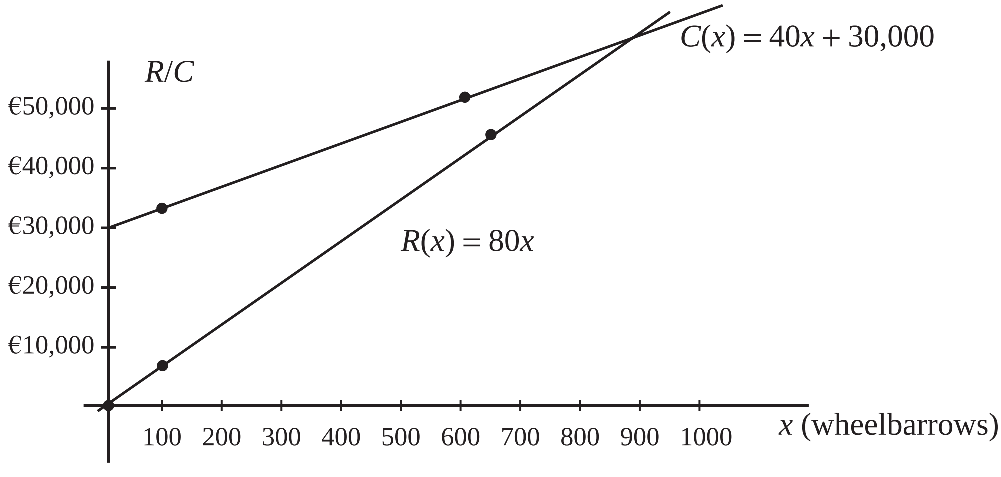
$$\Rightarrow \quad 30{,}000 = 6x$$

$$x = 5000 \text{ wheelbarrows}$$

(iv) Selling price = €80 per wheelbarrow

$\Rightarrow$ Revenue = €80x

(v)



(vi) To make a profit $80x > 40x + 30{,}000$

$$\Rightarrow 40x > 30{,}000$$

$$x > \frac{30{,}000}{40} = 750$$

The minimum number of wheelbarrows = 751.

(vii) Profit €$p = 80x - [40x + 30{,}000]$

€$p = 40x - 30{,}000$

Q7.

Number in queue	skipping two (a) number admitted first	skipping three (b) number admitted first
4	2	1
5	3	2
6	2	3
7	3	4
8	4	2
9	3	3
10	4	4
11	5	5
12	4	3
13	5	4
14	6	5
15	5	6.

(a) 4, 5, 6, (7), 8, 9, (10), 11, 12, (13), 14, 15.....70 → number in queue

number pattern $\underbrace{2, 3, 2}_{⑦}, \underbrace{3, 4, 3}_{⑩}, \underbrace{4, 5, 4}_{⑬}, \underbrace{5, 6, 5}_{16}$ → number admitted before seen.

⇒ (7), (10), (13), 16,

⇒ three numbers $x + (x+1) + x = 70$

$$3x + 1 = 70$$

$$3x = 69$$

$$x = 23$$

∴ 23, 24, 23

∴ number admitted first = 23 if there are 70 in queue

(note other patterns may also be found)

(b) 4, 5, 6, 7, 8, 9, (10), 11, 12, 13, (14), 15…

number pattern $\underbrace{1, 2, 3, 4}_{10}, \underbrace{2, 3, 4, 5}_{14}, \underbrace{3, 4, 5, 6}_{18}\ldots$ → number admitted before seen.

∴ $x + (x+1) + (x+2) + (x+3) = 70 + 2.$

$$4x + 6 = 72$$

$$4x = 66$$

$x = 18\frac{1}{2}$ not a whole number

$\therefore$ try $x+(x+1)+(x+2)+x=70+2$

$$4x+3=72$$

$$4x=69$$

$x=17\,{}^1\!/_4$ not a whole number

$\therefore$ try $x+(x+1)+(x-1)+x=70+2$

$$4x=72$$

$$x=18$$

$\therefore$ $18, 19, 17, 18$

$\Rightarrow$ the number admitted first $=18$