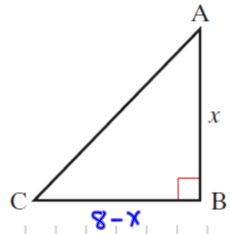
4. In the given right-angled triangle ABC, the lengths of [AB] and [BC] vary such that their sum is always 8 cm.

(i) If $|AB| = x$, express $|BC|$ in terms of x.

(ii) Find the maximum area of the triangle ABC.



(i) If $[AB] + [BC] = 8$
Since $[AB] = x$ $\Rightarrow$ $|BC| = 8 - x$

(ii) Max. Area?

$A = \frac{Bh}{2}$ $\qquad A = \frac{(x)(8-x)}{2} = 4x - \frac{1}{2}x^2$

$\frac{dA}{dx} = 0$ when A is max $\qquad \frac{dA}{dx} = 4 - x$

when $\frac{dA}{dx} = 0$ $\qquad 4 - x = 0 \Rightarrow x = 4$ cm

Area max? $\qquad A_{max} = 4(4) - \frac{1}{2}(4)^2 = 16 - 8$

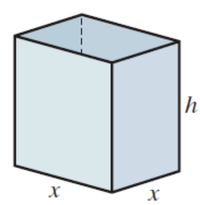
$A_{max} = 8 \text{ cm}^2$

5. A storage tank in the shape of a cuboid has a capacity of 108 m³. It has a square base of side x metres with vertical sides and open at the top.

(i) Express the height, h, in terms of x.

(ii) Show that the surface area, S, is given by $S = x^2 + \frac{432}{x}$.

(iii) Find the dimensions of the tank if the surface area is to be a minimum.



(i) Capacity = Volume

$V = LBH$ $\Rightarrow$ $(x)(x)(h) = 108 \Rightarrow \boxed{h = \frac{108}{x^2}}$

(ii) Surface Area?
note: 5 sides!

$SA = 1LB + 2LH + 2BH$ $\qquad SA = x^2 + 2hx + 2hx = x^2 + 4hx$

Sub. in $h = \frac{108}{x^2}$ $\qquad SA = x^2 + 4\left(\frac{108}{x^2}\right)x = x^2 + \frac{432}{x}$ QED

(iii) Dimensions if SA is min.?

when SA = min, $\frac{d\,SA}{dx} = 0 \Rightarrow$ $\qquad \frac{d\,SA}{dx} = 2x - 432x^{-2} = 0$

multiply by LCD ie x^2 $\qquad \Rightarrow 2x^3 - 432 = 0 \Rightarrow 2x^3 = 432$

$\Rightarrow x^3 = 216 \Rightarrow x = \sqrt[3]{216} \Rightarrow x = 6$

when $x = 6$, $h = ?$ $\qquad h = \frac{108}{x^2} = \frac{108}{(6)^2} \Rightarrow h = 3$

9. A closed cylindrical can has height h cm and radius r cm.
If the total surface area is 24π cm^2, find an expression for the volume, V cm^3, in terms of r.
Hence, find the value of r which will make the volume a maximum.

[**Note:** The surface area of a closed cylinder is $2\pi r^2 + 2\pi rh$.]

Volume in terms of R?

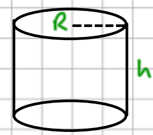
$$\boxed{V = \pi R^2 h}$$

$$TSA = \underbrace{2\pi R h}_{CSA} + \underbrace{2\pi R^2}_{2\,\text{DISCS}}$$

$$\boxed{TSA = 2\pi R(h+R)}$$

Volume?

When $V = \max$ $\Rightarrow \frac{dV}{dR} = 0$



$TSA = 24\pi$ cm^2

$$\Rightarrow 24\cancel{\pi} = 2\cancel{\pi}R(h+R)$$

$$\frac{12}{R} = h + R \Rightarrow \boxed{h = \frac{12}{R} - R}$$

$$V = \pi R^2 h \Rightarrow V = \pi R^2\left[\frac{12}{R} - R\right]$$

$$\Rightarrow \boxed{V = 12\pi R - \pi R^3}$$

$$\frac{dV}{dR} = 12\pi - 3\pi R^2 = 0$$

$$\Rightarrow 12\cancel{\pi} = 3\cancel{\pi}R^2$$

$$\Rightarrow R^2 = 4$$

$$\Rightarrow \boxed{R = 2\text{ cm}}$$