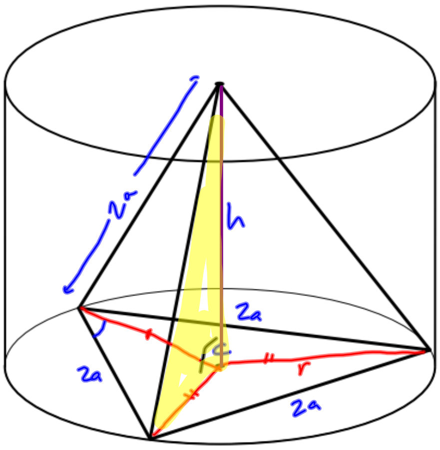
Question 9 **(25 marks)**

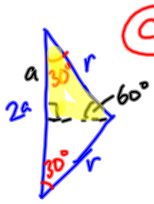
A regular tetrahedron has four faces, each of which is an equilateral triangle.

A wooden puzzle consists of several pieces that can be assembled to make a regular tetrahedron. The manufacturer wants to package the assembled tetrahedron in a clear cylindrical container, with one face flat against the bottom.

If the length of one edge of the tetrahedron is $2a$, show that the volume of the smallest possible cylindrical container is $\left(\frac{8\sqrt{6}}{9}\right)\pi a^3$.

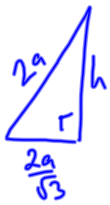


$\boxed{V = \pi R^2 h}$



CAH

$\cos 30° = \frac{a}{r} \Rightarrow r = \frac{a}{\cos 30°} = \frac{2a}{\sqrt{3}}$



$\boxed{A^2 = b^2 + c^2}$

$(2a)^2 = h^2 + \left(\frac{2a}{\sqrt{3}}\right)^2$

$4a^2 = h^2 + \frac{4a^2}{3} \Rightarrow h^2 = 4a^2 - \frac{4}{3}a^2$

$h^2 = \frac{8}{3}a^2 \Rightarrow h = \frac{2\sqrt{6}}{3}a$

$V = \pi\left(\frac{2a}{\sqrt{3}}\right)^2\left(\frac{2\sqrt{6}}{3}a\right) = \pi\left(\frac{4a^2}{3}\right)\left(\frac{2\sqrt{6}}{3}a\right)$

$V = \left(\frac{8\sqrt{6}}{9}\right)a^3\pi$ QED